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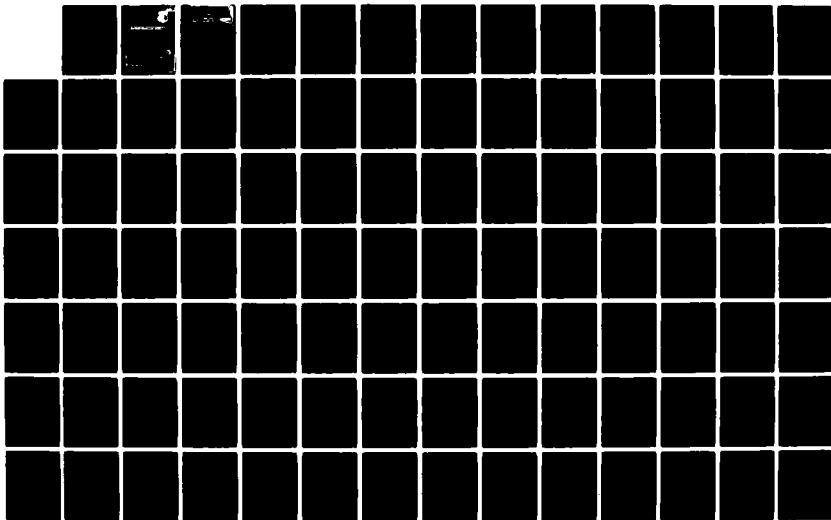
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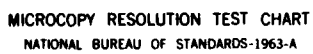
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## 20. ABSTRACT (CONTINUED)

to the experimenter in terms of removing nonsignificant factors and thus reducing the size of the experimental space.

This report documents the second phase of design, development, and use of interactive computer program to aid in the development of fractional factorial experimental designs. Fractional factorial experiments are a special class of experimental procedures that allow the user to perform a smaller number of experiments than would be required in the usual experimental procedures and which maximize information return while minimizing the number of observations (tests) required. The overall experimental design philosophy is described and a brief introduction into the theory of experimental design is presented. The Appendix describes how the computer program was constructed and how it should be used.

AED: Version II includes a mixed level capability in that one set of factors can be set at two levels and a second set of factors can be set at three levels in the same experimental (test) design. Also included is a Central Composite design capability with all factors at five levels.

This version updates and supersedes Version I that was published as AFAMRL-TR-81-100.

## PREFACE

This program was developed for the Air Force Aerospace Medical Research Laboratory under Contract F33615-79-C-0505. Dr. Robert G. Mills, AFAMRL, was the Air Force Program Manager. Mr. Edwin G. Meyer was the SDC Program Manager. Mr. William Rickels and Mrs. C. Hoyland were responsible for algorithm implementation and programming; as well as writing the report.

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## GLOSSARY

ALIAS	Effect that cannot be distinguished from another effect.
ALPHA	In a central composite design the non-zero coded level value of a factor at an axial point.
AXIAL POINTS OR STAR POINTS	In a central composite design for each factor there are two corresponding axial points: The given factor has coded level $-\text{ALPHA}$ at one point and $+\text{ALPHA}$ at the other, whereas all other factors have coded level zero at these points.
CENTER POINT	The point in a central composite design where all $N$ factors have coded level zero.
CENTRAL COMPOSITE DESIGN	A combination of a full or fractional two-level factorial design and some additional experimental points selected in a particular manner to allow the determination of the quadratic one factor effects. It is specifically intended to allow determination of the constraints used in defining a quadratic approximation of the response surface.
CODED LEVEL	The level of a factor translated from the true quantitative level used for simplifying calculations.

**CONFOUNDING**

An experimental arrangement in which certain effects cannot be distinguished from others.

**CORRELATION COEFFICIENT  
(Pearson R)**

The square root of the proportion of total variation accounted for by linear regression.

**CORRELATION INDEX R**

The square root of the proportion of total variation accounted for by the regression equation of the degree being fitted to the data.

**DEFINING CONTRAST**

Selection of effects to be confounded.

**DEGREES OF FREEDOM**

One less than the number of values required to compute the sum of squares.

**EFFECT**

Change in response caused by a change in the level of a factor.

**EXPERIMENT MODEL**

Hypothesized equation to describe the response as a function of the treatment.

**EXPERIMENTAL TRIAL**

One unit of a complete experiment, conducted with factors at levels defined by a single observation vector.

**FACTORIAL EXPERIMENT**

An experiment in which all levels of each factor in the experiment are combined with all levels of every other factor.

**FRACTIONAL FACTORIAL**

An experimental design in which only a fraction of a complete factorial is run.

**INTERACTION**

An interaction between two factors means that a change in response between levels of one factor is not the same for all levels of the other factor.

**MEAN SQUARE ERROR**

Sum of squares of the error divided by the number of degrees of freedom for the error term.

**MIXED LEVEL DESIGN**

A full or fractional factorial design where some factors of the design have a different number of levels than other factors of the design.

**OBSERVATION VECTOR**

Planned level of each factor for a single experimental trial.

**REAL WORLD LEVEL**

The true quantitative level of a factor that corresponds to a coded level.

**REGRESSION**

Linear - Response =  $A \cdot X_1 + B \cdot X_2 + C \cdot X_3 + \dots + Z \cdot X_N$ ; Quadratic - Response  
=  $A \cdot X_1 + B \cdot X_2 + \dots + C \cdot X_1 X_2 + D \cdot X_1 X_3 + \dots + E \cdot X_1^2 + F \cdot X_2^2 + \dots$

**REPLICATE**

Repetition of observation vectors applied to multiple experimental trials.

## RESPONSE FUNCTION

The function  $F$  or  $Y = F(X_1, X_2, \dots, X_N)$  where the levels of the factors are  $X_1, X_2, \dots, X_N$  and the response is  $Y$ .

## RESPONSE SURFACE

The surface in  $N+1$  dimensional space represented by the equation  $Y = F(X_1, X_2, \dots, X_N)$ .

## ROOT SUM SQUARE (RSS)

The square root of the sum of the squares represented by the formula:

$$\sum_{i=1}^N x_i^2$$

## ROTATABLE DESIGN

A central composite design that leaves the variance of the estimated response to be approximately constant throughout the sphere of radius one.

## R-SQUARED

Small  $r$ -Squared--refer to Correlation Coefficient  
Big  $R$ -Squared--refer to Correlation Index.

## TRIAL

A single set of factor values applied to the experimental subject for which the response is measured.

## INTRODUCTION

The Air Force Aerospace Medical Research Laboratory (AFAMRL) is engaged in the use of human operators to perform critical systems evaluation. The size and complexity of the various systems preclude the detailed analysis that would enable AMRL to examine each aspect of every system. Large numbers of factors (independent variables) are commonly encountered in real-world simulation or field problems. Complete full factorial experimental designs for problems involving large numbers of factors (20 factors are not uncommon) are very costly in time, manpower, and other test resources.

The use of fractional factorial designs permits the experimenter to employ sequential experimental design techniques. See Cochran and Cox (1957) and other references for a complete discussion of fractional factorial experimental designs. In this procedure, the various factors are examined and a potentially significant subset is defined. By using the proper aliasing of effects, a small fractional factorial experiment can be conducted. If additional effects/interactions are identified as being highly significant or if additional interactions are required to be examined, a larger fractional factorial design can be constructed by removing some of the aliasing requirements. This process of designing an experiment, data analysis, and design refinement is the basis of sequential experimental design.

Examples of multivariable design problems can be found in many Air Force and other R&D programs, e.g., Aume, Mills, et al., 1977. AMRL has been studying these experimental design problems for a number of years, including the studies performed by Simon (1973), Mills & Williges (1973), and Williges & Mills (1973, 1979) relating to human factors experimentation. This report represents an effort to implement some of the design strategies previously proposed.

Human factors experimentation is an especially critical area of research because the experimenter must consider the factors in the system being studied and the variations introduced by the presence of a human subject. To overcome these perturbations, the experimental procedure must be run many times with several different subjects to remove effects caused by the subjects and to

identify variations caused by the parameters being studied. Since this procedure requires a large number of experimental trials (e.g., observations, tests, etc.), it may not be feasible to conduct a study because of cost and time. One way to overcome this problem is to employ a set of experimental designs called fractional factorial experimental designs. Fractional factorial experiments are a special class of reduced data collection designs that allow the user to perform a smaller number of observations than would be required in the usual experimental procedures.

This effort provides the reader with an automated tool to design fractional factorial experiments. A tape of the User-Assisted Automated Experimental (Test) Design Program (AED): Version II source listing in FORTRAN 4-Plus can be obtained from AFAMRL/HEF, Attn: Dr. Robert G. Mills, WPAFB, OH 45433. For the purposes of this report, the authors assume that the reader possesses at least a conceptual knowledge of symmetrical experimental design procedures including fractional factorials. This assumption also holds true for the user of the initial version of the computer program which is being described. However, a long range objective of this effort is to eventually develop the program to the extent that, via the interactive mode, the program's user need have only a minimum knowledge of experimental design computational procedures. The primary intent is to develop the computer program such that it can be readily applied by the engineering, etc., community that is involved with performing simulator and live testing of systems. It should also be noted that although the computer program presented herein is designed to assist the sequential experimental design process (i.e., a series of experiments), it can also be used to create a "one-shot" experimental design.

This report provides background on experimental designs and the mathematical formulations implemented in the computer program. A discussion of how an experiment should be conducted is contained in the Philosophy of Experimental Design section. The class of experimental designs known as fractional factorial designs is described along with the terminology involved, the concept of aliasing, the evaluation of designs, and means of defining basic experimental blocks. Screening designs, response surface designs, polynomial approximations to the response surface, central composite designs, and mixed level designs are also

discussed. Brief commentaries on data collections, redesign, and irregular fractional factorial experiments are provided. Some predefined fractional factorial designs including optional aliasing selection to reduce aliasing of main and first-order effects are presented. A selected bibliography of books and reports that present more detailed information on these topics is given in the reference section. An appendix provides a detailed step-by-step description of the computer program.

This report is an update of a previous report and indicates the present status of the automated experimental design program (AED). The AED program contains adequate instructions and text to guide the user in its operation without the assistance of this report. It is provided as an aid to understanding the mathematical formulations and as a source of additional examples (Appendix). For the current version of the program, the following is a summary of its present capabilities.

1. Basic full or fractional designs where
  - (a) 2 level designs can have up to 20 factors with a maximum of 256 experimental trials.
  - (b) 3 level designs can have up to 12 factors with a maximum of 243 experimental trials.
  - (c) 5 level designs can have up to 8 factors with a maximum of 125 experimental trials.
2.  $2^K \times 3^L$  mixed level designs where
  - (a) The combined number of factors for both levels must be less than or equal to 20 ( $K + L \leq 20$ ) with a maximum of 256 total experimental trials.
  - (b) Experimental plans for combining the separate levels have been developed for 1/2, 1/3, 1/4, 1/6, 1/8, 1/9, 1/12, 1/16, 1/18, and 1/24 fractionations.



3. Rotatable and non-rotatable central composite designs.
4. 22 predefined stored designs for 2 levels.  
19 predefined stored designs for 3 levels.
5. Assistance in generating realizable 2 level designs for 1/2, 1/4, 1/8, and 1/16 fractionations.

### THE PHILOSOPHY OF EXPERIMENTAL DESIGN

The basis for an experimental design philosophy consists of six steps:

1. Problem recognition and initial study
2. Preliminary model definition
3. Data collection plan development
4. Data collection
5. Data analysis
6. Analysis of results and model reformulation.

In the first step, the experimenter recognizes the existence of a problem. He begins a preliminary study to identify the problem bounds and its associated parameters. This initial study provides a crude model of the system. In the second step, the experimenter examines this preliminary model and identifies those features that severely affect the performance of the system. He designs a data collection plan that enables him to test the previously hypothesized significant features. Without an adequate data collection plan, the experimenter may arrive at erroneous conclusions.

Once the data collection plan (called the experimental design) is complete, the experimenter "collects" the data. After the data are collected, data analysis is performed. Data analysis consists of the standard analysis methods, e.g., analysis of variance (ANOVA) techniques or regression analysis, if all factors are quantitative. This analysis identifies those factors that account for most of the system variation. According to Pareto's Principle, 80 percent of the variation in a system can be attributed to 20 percent of the factors.

After the data analysis is performed, the experimenter redesigns or refines his system model based on the results of the previous experimentation. This cycle of redesign, data collection, and analysis continues until the experimenter is satisfied with the accuracy of his results. At this point, he draws conclusions about the system based upon the experimentation.

### THE NEED FOR DESIGNED EXPERIMENTS

An experiment is conducted to provide information. An experimenter needs information to identify problem areas, to identify important factors, and to quantify responses. He obtains this information by collecting data. After problem definition is complete, the first step in an experiment is to define questions that need to be answered. Once the questions are identified, an experiment can be designed to aid in answering those questions. The key issue is that an experimenter must design his experiment before any data are collected.

The designer of an experiment must consider the statistical accuracy and the cost of the experiment. Statistical accuracy involves the proper selection of the response to be measured, determination of the number of factors that influence the response, the selection of the subset of these factors to be studied in the experiment being planned, the number of times the basic experiment should be repeated, and the form of the analysis to be conducted.

The cost of an experiment includes, among many other factors, expense incurred by running a single experimental condition (observation), analyzing the data, failing to meet a deadline, availability of subjects, and most importantly, perhaps drawing incorrect conclusions from the experiment. Although cost as a factor is not often discussed in the literature, it is at least as important as considerations of statistical accuracy. In an attempt to minimize the cost of an experiment, the designer usually attempts to choose the simplest experimental design possible, and to use the smallest sample size consistent with satisfactory results. Fortunately, most simple experimental designs are both statistically efficient and economical, so that the designer's efforts to obtain statistical accuracy usually result in economy.

## EXPERIMENTAL MODEL

The experiments being studied in a factorial experiment are called fixed effect models. The term fixed effect is related to the predefined levels that the various factors may assume. Consider two factors, A and B, which are being studied where there are  $N_A$  levels for treatment A and  $N_B$  levels for treatment B. The response in a two-factor experiment may be described by the model:

$$\begin{aligned}X_{ij} &= \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ij} \\i &= 1, 2, \dots, N_A \\j &= 1, 2, \dots, N_B\end{aligned}$$

where

$$\begin{aligned}\mu &= \text{overall mean effect} \\ \alpha_i &= \text{true effect of the } i\text{th level of factor A} \\ \beta_j &= \text{true effect of the } j\text{th level of factor B} \\ (\alpha\beta)_{ij} &= \text{effect of the interaction between } \alpha_i \text{ and } \beta_j \\ \epsilon_{ij} &= \text{experimental error}\end{aligned}$$

A similar model for three factors may be written as:

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijk}$$

The assumptions in a full factorial experiment allow for the examination of each main effect and all interactions. A fractional factorial experiment assumes that the high-order interactions are insignificant. For example, in the three-factor model, if the assumption is made that the interactions  $(\alpha\gamma)$ ,  $(\beta\gamma)$ , and  $(\alpha\beta\gamma)$  are insignificant, the model becomes:

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

This permits fewer experimental observations to determine the relative significance of the remaining terms in the model. The effects considered to be insignificant are included in the model error term.

This experimental model can be evaluated using the standard analysis of variance (ANOVA) techniques or a regression analysis may be run, if all factors are quantitative, to determine regression coefficients.

#### NOMENCLATURE--NOTATION AND TERMINOLOGY

The previous section showed that responses could be modeled as equations involving true effects of each factor at the level involved, the effects of the interactions among factors, the overall mean effect, and the true test (experimental) error. The techniques for manipulating response data from individual experimental trials to arrive at estimates of the values for each of the terms in the mathematical model involve consideration of response values for various combinations of factors and levels. Two standard means of notation are used to represent these response values. These are illustrated in the following example.

Consider an experiment involving three factors with each factor having two possible levels. If the factors are represented by a, b, and c, and the levels, by 0 and 1, the possible trials and notations used to represent the responses are shown in Table 1.

Table 1. Full Factorial, Three-Factor, Two-Level Experiment

EXPERIMENTAL TRIAL (FACTOR AND LEVEL)	EFFECT OR INTERACTION	NOTATION
$a_0b_0c_0$	I	000
$a_0b_0c_1$	C	001
$a_0b_1c_0$	B	010
$a_0b_1c_1$	BC	011
$a_1b_0c_0$	A	100
$a_1b_0c_1$	AC	101
$a_1b_1c_0$	AB	110
$a_1b_1c_1$	ABC	111

Main effects are represented by those trials whose notation has a nonzero value in only one column of the notation. Two-factor, or first-order interactions, are represented by those trials whose notation has a nonzero value in two columns. Higher-order interactions are represented by those trials whose notation has a nonzero value in more than two columns. In this example, main effects are A, B, and C. First-order interactions are AB, AC, and BC. The only higher-order interaction is ABC.

It is apparent from Table 1 that the notation consists simply of the subscripts representing the levels of the factors in sequential order (thus, trial a0b1c1 has a response notation of 011). The effect or interaction response is represented by the sequence of the factors raised to the power of the level involved (thus, trial a0b1c1 results in the interaction response  $A^0B^1C^1 = BC$ ).

In a similar manner, effects and notations can be defined for the three-level case. Consider an experiment involving three factors with each factor having three possible levels. If the factors are represented by a, b, and c, and the levels by 0, 1, and 2, the possible trials and notations used to represent the responses are shown in Table 2. Note that the main effects are C,  $C^2$ , B,  $B^2$ , A, and  $A^2$  because the notation for these trials has a nonzero value in only one column of the notation.

#### FRACTIONAL FACTORIAL EXPERIMENTS

A full factorial experimental design involves an experiment in which every level of each factor is combined with every level of every other factor. If an experiment has N factors and each factor may assume one of P levels, there is a total of  $P^N$  combinations.

Table 3 is an example of an experiment with three factors at two levels. This example is taken from an AMRL study of the MISVAL program. The term "MISVAL" designates the Missile Launch Envelope Technology Development Program being conducted by the Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base, Ohio. The definitions of factors and levels used in the examples are not considered necessary in order to convey the intent of the examples.

Table 2. Full Factorial, Three-Factor, Three-Level Experiment

EXPERIMENTAL TRIAL (FACTOR AND LEVEL)	EFFECT OR INTERACTION	NOTATION
$a_0b_0c_0$	I	000
$a_0b_0c_1$	C	001
$a_0b_0c_2$	$C^2$	002
$a_0b_1c_0$	B	010
$a_0b_1c_1$	BC	011
$a_0b_1c_2$	$BC^2$	012
$a_0b_2c_0$	$B^2$	020
$a_0b_2c_1$	$B^2C$	021
$a_0b_2c_2$	$B^2C^2$	022
$a_1b_0c_0$	A	100
$a_1b_0c_1$	AC	101
$a_1b_0c_2$	$AC^2$	102
$a_1b_1c_0$	AB	110
$a_1b_1c_1$	ABC	111
$a_1b_1c_2$	$ABC^2$	112
$a_1b_2c_0$	$AB^2$	120
$a_1b_2c_1$	$AB^2C$	121
$a_1b_2c_2$	$AB^2C^2$	122
$a_2b_0c_0$	$A^2$	200
$a_2b_0c_1$	$A^2C$	201
$a_2b_0c_2$	$A^2C^2$	202
$a_2b_1c_0$	$A^2B$	210
$a_2b_1c_1$	$A^2BC$	211
$a_2b_1c_2$	$A^2BC^2$	212
$a_2b_2c_0$	$A^2B^2$	220
$a_2b_2c_1$	$A^2B^2C$	221
$a_2b_2c_2$	$A^2B^2C^2$	222

Table 3. MISVAL Example

LABEL	FACTOR	LOW LEVEL (0)	HIGH LEVEL (1)
A	MLE CONCEPT	FAAC CONCEPT	GD CONCEPT
B	PILOT FUNCTION	FUNCTION 1	FUNCTION 2
C	MISSILE TYPE	AIM-7F	AIM-9F

The  $2^3 = 8$  combinations of these factors that would comprise a full factorial experiment for the MISVAL program are given in Table 4.

Table 4. Full Factorial Experiment

EXPERIMENTAL UNIT	LABEL	NOTATION	MLE CONCEPT	PILOT FUNCTION	MISSILE TYPE
1	I	000	FAAC	1	AIM-7F
2	C	001	FAAC	1	AIM-9F
3	B	010	FAAC	2	AIM-7F
4	BC	011	FAAC	2	AIM-9F
5	A	100	GD	1	AIM-7F
6	AC	101	GD	1	AIM-9F
7	AB	110	GD	2	AIM-7F
8	ABC	111	GD	2	AIM-9F

Table 5 shows the number of observations required in a full factorial experiment for experiments with 2 to 10 factors at 2 or 3 levels. Note that the number of observations required rises drastically as the number of factors and/or levels increases.

A full factorial experimental design provides an estimate of every possible effect, i.e., one is able to estimate those effects caused by all combinations of factors. In many experiments, interactions among factors may be insignificant. Interactions involving two factors are called first-order interactions, and interactions among three factors are called second-order interactions.

Table 5. Full Factorial Experiment Size

N NUMBER OF FACTORS	P = 2 LEVELS PER FACTORS	P = 3 LEVELS PER FACTORS
2	4	9
3	8	27
4	16	81
5	32	243
6	64	729
7	128	2187
8	256	6561
9	512	19683
10	1024	59049

In many human factors experiments, the assumption that second and higher-order interactions are insignificant is reasonable (Simon, 1973).

The total number of effects and interactions is given by  $P^N - 1$ . The number of main effects is given by  $(P - 1)N$ . The number of first-order interactions is given by:

$$\frac{(P-1)^2}{2} N(N-1)$$

Thus, the number of higher-order interactions is given by:

$$P^N - 1 - (P - 1)N - \frac{(P - 1)^2}{2} N(N - 1)$$

Table 6 shows the number of main, first-order, and higher-order effects for a variety of factorial experiments.

Figure 1 shows examples of the groupings of main, first-order, and higher-order effects for three factors at two and three levels.



Table 6. Effect/Interaction Summary

N NUMBER OF FACTORS	P = 2			P = 3		
	LEVELS PER FACTOR			LEVELS PER FACTOR		
	MAIN EFFECTS	1ST ORDER	HIGHER ORDER	MAIN EFFECTS	1ST ORDER	HIGHER ORDER
2	2	1	0	4	4	0
3	3	3	1	6	12	8
4	4	6	5	8	24	48
5	5	10	16	10	40	192
6	6	15	42	12	60	656
7	7	21	99	14	84	2088
8	8	28	219	16	112	6432
9	9	36	466	18	144	19520
10	10	45	968	20	180	58848

A fractional factorial design, sometimes called a fractional replication, is a portion or a fraction of a complete factorial experiment. In a fractional replicate, certain interactions cannot be separated from other interactions. This is the price that is paid for reducing the number of experimental trials. Interactions or effects that cannot be separated are said to be aliased or confounded.

The use of fractional factorial experiments is based on the assumption that higher-order interactions above first-order are insignificant and need not be examined in detail. For example, consider an experiment in which main effect A is aliased with interaction BCD. When data are collected, the experimenter estimates the response caused by effect A and BCD together. There is no way to know if the response is due only to A or if effect BCD plays a significant part in the response. Thus, effects A and BCD are not separable. The assumption in a fractional factorial experiment is that the contribution caused by BCD would be negligible.

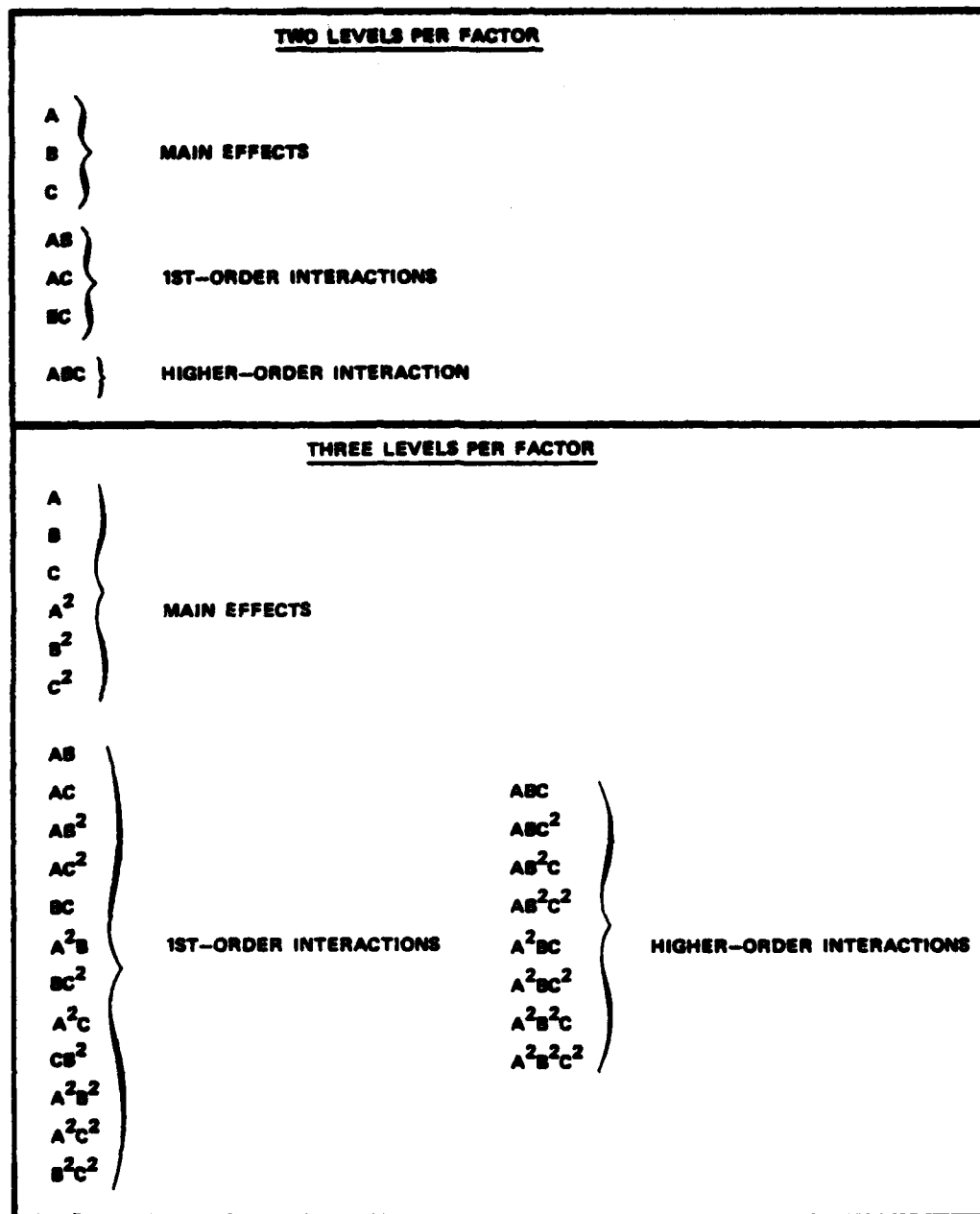


Figure 1. Main Effects and Interactions

A full factorial experiment is useful when an experimenter requires that:

1. Every main effect of every factor be estimated independently of every other one.
2. The dependence of the effect of every factor upon the levels of the others (the interactions) be determined.
3. The effects be determined with maximum precision.

If an experimenter does not require this level of detail, or if faced with time or budget limitations that prohibit a full factorial experiment, fractional factorial designs are available. The primary assumption in the use of a fractional factorial experiment is that higher-order interactions are insignificant. If this assumption is not valid for a particular experiment, a fractional factorial design should not be used. In most human factors experiments, however, this is a reasonable assumption and can result in a significant reduction in the number of experimental (test) trials or observations required.

Interactions that are assumed to be insignificant can be used to define the aliasing or confounding used in the fractional factorial design. The concept of aliasing is discussed in the next section.

## ALIASING

### OVERVIEW

Each successive step in fractionating a full factorial design, or dividing it into blocks, requires that an additional effect referred to as a defining contrast be defined for the fractional factorial design. A defining contrast is a selected observation vector whose factor combinations will not be important to the experimentation. Defining contrasts are then used in generating the alias set. The alias set is composed of all factor combinations of the defining contrasts. Thus, a two-level, one-half design requires one defining contrast to be defined by the experimenter; while a one-fourth design requires two defining contrasts, and so on. The defining contrasts must be selected by the experimenter to meet the requirements of the design.

The selection of defining contrasts is important in the design of a fractional factorial experiment. In a given experiment, the value of aliased terms cannot be estimated; thus, no term of interest to the experimenter should be selected as a defining contrast.

Defining contrasts are usually selected to avoid aliasing main effects with other main effects. In a sequential design, however, it may be desirable to alias two main effects. For example, if the experimenter suspects that two factors, A and E, are not significant, he might design the first pilot experiment so that A and E, are confounded. If the data from this pilot experiment show that the estimated values of A and E are not significant, then the experimenter's suspicions are confirmed. These two factors can be dropped, thus reducing the experiment size for the next pilot experiment.

The defining contrasts can be described by identities that determine which effects will be confounded. The experimenter does not have a completely free hand in the selection of these defining contrasts. Defining contrasts are linearly independent if one defining contrast is not a factor combination of another. Unless the selected defining contrasts are linearly independent, some factor combination of one defining contrasts will be the same as a factor combination of the nonindependent defining contrasts selected. This indicates that the nonindependent defining contrasts is redundant, and the experimenter has selected fewer defining contrasts than planned. This results in a larger experiment block size than desired. In this case, it is necessary to redefine the nonindependent defining contrast so that a set of independent defining contrasts is selected.

The following paragraphs provide background for the generation of the alias set and development of alias summaries. These operations are performed within the computer program, and understanding of this material is not necessary to use the program.

#### OPERATIONS WITH ALIASES

Assuming each of N factors will be varied over P levels, the set of fractional factorial experiments considered here is the  $1/P^M$  designs, where M is a

positive integer. Thus two-level designs might be  $1/2$ ,  $1/4$ ,  $1/8$ ,  $1/16$ , etc. Three-level designs might be  $1/3$ ,  $1/9$ ,  $1/27$ ,  $1/81$ , etc. In general, a  $1/p^M$  design requires  $M$  defining contrasts. Specifying the defining contrasts is an important problem in designing fractional factorial experiments. One way to specify the defining contrasts is to describe which effects are to be confounded.

Although confounding two factor interactions with other two factor interactions is not always desirable, in a good experimental design, this will be done in the following examples for ease of calculation and demonstration. If effects  $AB$  and  $CD$  are to be confounded, the user may specify the defining contrast as  $AB = CD$ . Defining contrasts may also be described in terms of the identity effect,  $I$ . This is accomplished by multiplying both sides of the equation in this example by  $AB$ , yielding  $A^2B^2 = ABCD$ . Assuming a two-level problem, apply modulo 2 arithmetic to the exponents of the factors  $A^2B^2 = A^0B^0 = I = ABCD$ . If the effects to be confounded are  $A^2B = CD$  in a three-level problem, first multiply both sides by  $A^2B$  using module 3 arithmetic on the exponents. Modulo  $P$  is merely the remainder when the number is divided by  $P$  (e.g., 4 modulo 3 = 1, 12 modulo 3 = 0, 13 modulo 3 = 1, etc.).

This gives

$$\begin{aligned}(A^2B)(A^2B) &= A^2BCD \\ A^4B^2 &= AB^2 = A^2BCD\end{aligned}$$

Multiply both sides again by  $A^2B$ , giving  $(AB^2)(A^2B) = (A^2BCD)(A^2B)$

$$\begin{aligned}\text{or } A^3B^3 &= I = A^4B^2CD = AB^2CD \\ \text{or } I &= AB^2CD.\end{aligned}$$

Each effect will be aliased with  $(p^M - 1)$  other effects. For  $1/p^M$  designs,  $M$  must be less than or equal to  $(N - 1)$ . As the number of defining contrasts increases, the number of effects aliased with each effect increases rapidly as shown in Table 7.

Table 7. Number of Effects Aliases

DESIGN	P = NO. OF LEVELS	M	NO. OF DEFINING CONTRASTS	NO. OF EFFECTS ALIASED WITH EACH EFFECT
1/2	2	1	1	1
1/4	2	2	2	3
1/8	2	3	3	7
1/16	2	4	4	15
1/32	2	5	5	31
1/64	2	6	6	63
1/128	2	7	7	127
1/3	3	1	1	2
1/9	3	2	2	8
1/27	3	3	3	26
1/81	3	4	4	80
1/243	3	5	5	242
1/729	3	6	6	728
1/2187	3	7	7	2186

The number of effects aliased with each effect (such as a main effect) increases rapidly as smaller fractional factorial designs are considered. Thus, as the number of defining contrasts (M) increases without a corresponding increase in the number of factors (N), it becomes difficult to select defining contrasts that avoid aliasing main effects with other main effects in this case.

The total alias combination set may be generated by considering all combinations of all powers of the individual defining contrasts from 1 to the  $(P - 1)$ th power. Thus, if a two-level experiment is being considered ( $P = 2$ ), only the first power of the defining contrasts is considered in deriving the alias set. For a three-level experiment ( $P = 3$ ), both first and second powers of the defining contrasts are considered in deriving the alias set.

For example, consider a three-level experiment involving four factors ( $P = 3$ ,  $N = 4$ ). Suppose M is selected as a value of 3, resulting in a 1/27

design. From the tabulation, we find that three defining contrasts are required and each effect will be aliased with 26 effects or interactions. The alias set may be derived by representing all the integers from 1 to  $(p^M - 1)$  in base  $p$  arithmetic representation (1 to 26 in this example expressed in base 3 arithmetic).

1 = 001	10 = 101	19 = 201
2 = 002	11 = 102	20 = 202
3 = 010	12 = 110	21 = 210
4 = 011	13 = 111	22 = 211
5 = 012	14 = 112	23 = 212
6 = 020	15 = 120	24 = 220
7 = 021	16 = 121	25 = 221
8 = 022	17 = 122	26 = 222
9 = 100	18 = 200	

Each digit of the base 3 representation is used as the power to which each of the three defining contrasts is raised. For example, the combination 121 indicates that the first and third defining contrasts are raised to the first power while the second defining contrast is squared.

From this list, only those combinations that are in standard form are used, since the other combinations will result in duplications. A combination is in standard form if the leading nonzero exponent is 1. Thus, 120 is in standard form whereas 210 is not.

In our example ( $P = 3$ ,  $N = 4$ ,  $M = 3$ ), let us specify the defining contrasts selected as  $I = ABCD = B^2C^2D = A^2B$ . The tabulation of combinations is shown in Table 8. Note that the exponents are reduced modulo  $P$  to arrive at the final alias combinations. Thus,  $(A^2B)^2 = A^4B^2 = AB^2$  modulo 3.

In our example for our experiment design, we assigned three defining contrasts. The total alias set was then derived (26 in this case), which represents the combinations applicable to this design. The experimenter must now be concerned with how the individual effects and interactions are aliased.

Table 8. Total Alias Set

POWER SET	DEFINING CONTRAST COMBINATION	TOTAL ALIAS SET
001	$A^2B$	$A^2B$
002	$(A^2B)^2$	$AB^2$
010	$B^2C^2D$	$B^2C^2D$
011	$(B^2C^2D)(A^2B)$	$A^2C^2D$
012	$(B^2C^2D)(A^2B)^2$	$ABC^2D$
020	$(B^2C^2D)^2$	$BCD^2$
021	$(B^2C^2D)^2(A^2B)$	$A^2B^2CD^2$
022	$(B^2C^2D)^2(A^2B)^2$	$ACD^2$
100	$ABCD$	$ABCD$
101	$(ABCD)(A^2B)$	$B^2CD$
102	$(ABCD)(A^2B)^2$	$A^2CD$
110	$(ABCD)(B^2C^2D)$	$AD^2$
111	$(ABCD)(B^2C^2D)(A^2B)$	$BD^2$
112	$(ABCD)(B^2C^2D)(A^2B)^2$	$A^2D^2$
120	$(ABCD)(B^2C^2D)^2$	$AB^2C^2$
121	$(ABCD)(B^2C^2D)^2(A^2B)$	$C^2$
122	$(ABCD)(B^2C^2D)^2(A^2B)^2$	$A^2BC^2$
200	$(ABCD)^2$	$A^2B^2C^2D^2$
201	$(ABCD)^2(A^2B)$	$AC^2D^2$
202	$(ABCD)^2(A^2B)^2$	$BC^2D^2$
210	$(ABCD)^2(B^2C^2D)$	$A^2BC$
211	$(ABCD)^2(B^2C^2D)(A^2B)$	$AB^2C$
212	$(ABCD)^2(B^2C^2D)(A^2B)^2$	$C$
220	$(ABCD)^2(B^2C^2D)^2$	$A^2D$
221	$(ABCD)^2(B^2C^2D)^2(A^2B)$	$ABD$
222	$(ABCD)^2(B^2C^2D)^2(A^2B)^2$	$B^2D$



### ALIAS SET GENERATION IN TWO LEVEL DESIGNS

The alias set for a given fractional factorial design may be selected to meet the objectives of the experimenter within certain constraints. For example, if it is known that two factors, A and B, are not significant, the experimenter may choose to deliberately confound these effects and select one defining contrast as AB. Obviously, it is not possible to develop guidelines for every case which might arise. Since a large portion of possible experimentation is directed toward screening or the identification of significant factors, emphasis has been placed on examining the selection of aliases for isolation designs in which factors of interest are not confounded with other factors of interest so their significance can be determined.

Generally, in a two-level experiment involving N factors, it is desired to isolate all single factors and two factor interactions. This is under the assumption that three factor and higher order interactions are insignificant. For this situation to exist, all members of the alias set must contain a minimum of five factors. There will be cases in which this cannot be achieved (e.g., an experiment which involves fewer than five factors).

To obtain the largest degree of isolation for a given experiment, all members of the alias set should contain approximately the same number of factors that are at their high level.

Given: N = number of factors

M = fractionation measure (fraction =  $\frac{1}{2^M}$ )

S = number of members of alias set

Then:  $S = 2^M - 1$

$\Sigma$  = maximum number of factors which the summation of all members of the alias set can contain  
 $= 2^M - 1 - N$ .

To illustrate, consider a  $1/8$  fractionation of a  $2^5$  experiment.

Here N = 5, M = 3 ( $\frac{1}{2^3} = \frac{1}{8}$ )

$$S = 2^3 - 1 = 7 \text{ members of alias set}$$

$$\Sigma = 2^2 \times 5 = 4 \times 5 = 20 \text{ factors}$$

In this example, the seven members of the alias set will contain a maximum of 20 factors. The alias set can consist of either  $S$  members, each of which has an even number of factors, or  $\frac{S+1}{2}$  members containing an odd number of factors and  $\frac{S-1}{2}$  members containing an even number of factors. In our example, the total alias set may thus consist of either:

- (a) 7 members each containing an even number of factors. The summation should equal 20. An example would be five members of 2 factors, one member of 4 factors, and one member of 6 factors. This is represented as:

$$5(2) + (4) + (6) = 20$$

A better example for this case, in terms of having all members of the alias set with as close to the same number of factors as possible would be

$$4(2) + 3(4) = 20$$

- (b) 4 members containing an odd number of terms, and 3 containing an even number of terms. For our example, this would be

$$2(2) + 4(3) + 1(4) = 20.$$

For our sample case, the alias set design in (b) is better than that in (a) in that (b) only has two 2 factor members while (a) has four 2 factor members. The quality of an alias set design may be quantified by determining the root sum square (RSS) of the departure from the mean:

$$\text{Mean} = \frac{20 \text{ factors}}{7 \text{ members}} = 2.857 \text{ factors/member}$$

$$\text{Quality} = \sqrt{\frac{S}{2} \sum_{i=1}^S (x_i - \bar{x})^2}, \text{ where } x_i = \text{number of factors for the } i\text{th case}$$

$\bar{x}$  = means, and S = number of members of the alias set.

$$\text{Quality (a)} = 4(2-2.857)^2 + 3(4-2.857)^2 \quad 1/2 = 6.856 \quad 1/2 = 2.619$$

$$\text{Quality (b)} = 2(2-2.857)^2 + 4(3-2.857)^2 + 1(4-2.857)^2 \quad 1/2 = 2.856 \quad 1/2 = 1.690$$

Thus the members of alias set design (b) are clustered nearer the mean than the members of (a) are.

#### THEORETICAL "BEST" ISOLATION DESIGNS

From the rules in the preceding section, it is possible to formulate the "best" theoretical isolation designs. "Best," in this context, means that the members of the alias set contain as close to the same number of high level factors as possible. These theoretical designs are presented in Tables 9 through 20 for the range of values of M and N which are included in the two level design portion of the AED program.

Table 9. M = 1. 1 Member of Alias Set

N	2	3	4	5	6	7	8	9	10	11
2	1									
3		1								
4			1							
5				1						
6					1					
7						1				
8							1			
9								1		

Table 10.  $M = 2$ . 3 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
3	3									
4	1	2								
5		2	1							
6			3							
7			1	2						
8				2	1					
9					3					
10					1	2				

Table 11.  $M = 3$ . 7 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
4	6		1							
5	2	4	1							
6		4	3							
7			7							
8			3	4						
9			1	4	2					
10				3	3	1				
11				1	3	3				

Table 12.  $M = 4$ . 15 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
5	6	8	1							
6	2	8	5							
7		6	7	2						
8		2	7	6						
9			5	8	2					
10			1	8	6					
11				5	7	3				
12				1	7	7				

Table 13.  $M = 5$ . 31 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
6	6	16	9							
7		14	15	2						
8			29		2					
9			13	16	2					
10			5	16	10					
11				13	15	3				
12				5	15	11				
13					12	16	3			

Table 14.  $M = 6$ . 63 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
7			30	31	2					
8			14	31	18					
9				29	32	2				
10				13	32	18				
11					29	31	3			
12					13	31	19			
13						28	32	3		
14						12	32	19		

Table 15.  $M = 7$ . 127 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
8		30	63	35						
9			61	64	2					
10			29	64	34					
11				61	63	3				
12				29	63	35				
13					60	64	3			
14					28	64	35			
15						60	63	4		

Table 16. M = 8. 255 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
9		2	125	128			2			
10			61	128	66					
11				125	127	3				
12				61	127	67				
13					124	128	3			
14					60	128	67			
15						124	127	4		
16						60	127	68		

Table 17. M = 9. 511 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
10			125	256	130					
11				253	255	3				
12				125	255	131				
13					252	256	3			
14					124	256	131			
15						252	255	4		
16						124	255	132		
17							251	256	4	

Table 18. M = 10. 1023 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
11			48	462	462	50	1			
12				253	511	259				
13					508	512	3			
14					252	512	259			
15						508	511	4		
16						252	511	260		
17							507	512	4	
18							251	512	260	

Table 19. M = 11. 2047 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
12				509	1023	515				
13					1020	1024	3			
14					508	1024	515			
15						1020	1023	4		
16						508	1023	516		
17							1019	1024	4	
18							507	1024	516	
19								1019	1023	5

Table 20. M = 12. 4095 Members of Alias Set

N	2	3	4	5	6	7	8	9	10	11
13					2044	2048	3			
14					1020	2048	1027			
15						2044	2047	4		
16						1020	2047	1028		
17							2043	2048	4	
18							1019	2048	1028	
19								2043	2047	5
20								1019	2047	1029

To illustrate the use of the tables, consider the previous example with M = 3 and N = 5. In Table 11 headed M = 3, in the row for N = 5, we find the number 2 under the column headed "2," the number 4 under the column headed "3," and the number 1 under the column headed "4." This is read as

$$2(2) = 4(3) + 1(4)$$

The optimum isolation design for this case would result in an alias set containing two members consisting of two factors, four members consisting of three factors,

and one member consisting of four factors. Since the alias set contains two factor terms, it is not possible to isolate main effects.

From inspection of Tables 9 through 20, it can be seen that within the constraints of the AED program (number of trials  $\leq 256$ ) and realizability, for a given value of M, the number of factors must equal or exceed the number shown in the following table to permit isolating all main and two factor effects:

<u>M</u>	<u>N<math>\geq</math></u>
1	5
2	8
3	10
4	11
5	11
6	11
7	11
8	11
9	11
10	12
11	12
12	13

Alternatively, for a given value of N (number of factors), the allowable values of M are shown in the following table to permit complete isolation of main and two factor interactions:

<u>N</u>	<u>M</u>
2	Not possible
3	Not possible
4	Not possible
5	M = 1
6	M = 1
7	M = 1
8	M = 1,2



9	M = 1,2
10	M = 2,3
11	M = 3,4,5,6,7,8,9,10
12	M = 4,5,6,7,8,9,10,11
13	M = 5,6,7,8,9,10,11,12
14	M = 6,7,8,9,10,11,12
15	M = 7,8,9,10,11,12
16	M = 8,9,10,11,12
17	M = 9,10,11,12
18	M = 10,11,12
19	M = 11,12
20	M = 12

Similar tables could be constructed for isolation of main effects only (all members of alias set contain at least four factors), isolation of main, two factor, and three factor interactions (all members of the alias set contain at least six factors), etc.

#### REALIZABLE DESIGNS

All of the designs presented in the Tables 9 through 20 are not realizable, particularly in those cases in which M approaches N. However, inspection of the theoretically best isolation designs will show if the desired results in terms of isolation could be obtained if the design can be found. If the design is not adequate, perhaps more trials should be added (reduce M) to improve the isolation. If the table design is adequate, the next step is to determine if the design is stored in the AED program. The designs stored in the AED program are the best realizable isolation designs that have been found to date.

If the desired design does not reside in the AED program, a realizable design can be constructed following the procedure described in the next section. Variants of this procedure can also be used to modify stored designs to meet special needs of the experimenter.

### ALIAS SET MODIFICATIONS AND THE GENERATION OF NEW DESIGNS

The alias set is generated by developing all of the possible combinations of the defining contrasts. As an example, if three defining contrasts, ( $M = 3$ ), say AB, BC, and CD are used in a four factor problem ( $N = 4$ ), the resulting alias set is generated as follows:

1	Defining Contrast No. 1 = AB	1100	2 factors
2	Defining Contrast No. 2 = BC	0110	2 factors
1x2	Product of 1 and 2 = AC	1010	2 factors
3	Defining Contrast No. 3 = CD	0011	2 factors
1x3	Product of 1 and 3 = ABCD	1111	4 factors
2x3	Product of 2 and 3 = BD	0101	2 factors
1x2x3	Product of 1,2, and 3 = AD	1001	2 factors

This is a  $6(2) + 1(4)$  design

A new design for  $N = 5$  can be built from this design by adding another column. The goal should be to increase the two factor terms and not increase the four factor ( $1 \times 3$ ) term.

This can be done as follows:

<u>Contrast Product</u>	<u>Original Design</u>	<u>Added Column</u>	
1	1100	1	3 factors
2	0110	1	3 factors

<u>Contrast Product</u>	<u>Original Design</u>	<u>Added Column</u>	
12	1010	0	2 factors
3	0011	1	3 factors
13	1111	0	4 factors
23	0101	0	2 factors
123	1001	1	3 factors

This results in a  $2(2) + 4(3) + 1(4)$  design for  $N = 5$  which means the best theoretical design from Table 11 for  $N = 5$  and  $M = 3$ .

The defining contrasts for this design are:

11001 = ABE

01101 = BCE

00111 = CDE

Now assume we wish to build a design for  $N = 6$ . This can be done by generating another new column. The new column should increase the two factor terms ( $1 \times 2$  and  $2 \times 3$ ) and not increase the four factor term ( $1 \times 3$ ). This will result in the following design:

<u>Contrast Product</u>	<u>5 Factor Design</u>	<u>Added Column</u>	
1	11001	0	3 factors
2	01101	1	4 factors
12	10100	1	3 factors
3	00111	0	3 factors
13	11110	0	4 factors
23	01010	1	3 factors
123	10011	1	4 factors

This results in  $4(3) + 3(4)$  design, which again is a best isolation design. In general, as  $M$  increases, this procedure will not result in best isolation designs, but all designs will be achievable.

The process can be used in reverse by removing columns. In fact, a poor design can be improved by removing columns, then adding columns by paying attention to the numbers of factors in the rows. To illustrate, assume the following ten factor design is available:

<u>Contrast Product</u>	<u>10 Factor Design</u>	
1	1111110000	6 factors
2	0001111100	5 factors
12	1110001100	5 factors
3	1101010011	6 factors
13	0010100011	4 factors

23	1100101111	7 factors
123	0011011111	7 factors

This is a  $1(4) + 2(5) + 2(6) + 2(7)$  design. The first step is to reduce this to a nine factor design by removing a column. Columns 3,5,9, and 10 should not be removed since this would reduce the four factor member ( $1 \times 3$ ). In the same manner, columns 3,4, and 6 should not be removed since the seven factor term ( $2 \times 3$ ) would not be reduced. Also columns 1, 2, and 5 should not be removed since this would not reduce the seven factor term  $1 \times 2 \times 3$ . This leaves columns 7 and 8 as candidates for removal. Removing column 7, we are left with a nine factor design. A new column can then be added as shown, which now results in a best ten factor design.

<u>Contrast Product</u>	<u>Resulting 9 Factor Design</u>	<u>Factors</u>	<u>Added Column</u>	<u>Factors</u>
1	111111000	6	0	6
2	000111100	4	1	5
12	111000100	4	1	5
3	110101011	6	1	7
13	001010011	4	1	5
23	110010111	6	0	6
123	001101111	6	0	6

The nine factor design is a  $3(4) + 4(6)$  design. The new ten factor design is a  $3(5) + 3(6) + 1(7)$  design as compared with our starting point of a  $1(4) + 2(5) + 2(6) + 2(7)$  design. Thus, we have arrived at a much improved design.

### ALIAS SUMMARY

The primary concern of the experimenter is directed toward examining how all main effects and first-order interactions are aliased. The aliasing for any effect can be found by multiplying the effect by the total alias set (applying modulo P arithmetic to the factor exponents).

For example, consider a design consisting of 6 factors each at 2 levels with an M of 2. This is a one-fourth design, and two defining contrasts must be specified by the experimenter. Assume that the identities selected are  $I = ABCE = ABDF$ . Following the rules given in the previous sections, we find that each effect is aliased with three effects; and the complete alias set consists of ABCE, ABDF, and CDEF. The main effects and first-order interactions and how they are aliased are illustrated in Table 21.

Individual effects or interactions can be examined by multiplying the particular effect by each member of the alias set. For example, with  $I = ABCE = ABDF = CDEF$ , the interaction EF can be found to be aliased as:

$$\begin{aligned}(I)(EF) &= (ABCE)(EF) = (ABDF)(EF) = (CDEF)(EF) \\ EF &= ABCE^2F = ABDEF^2 = CDE^2F^2 \\ EF &= ABCF = ABDE = CD\end{aligned}$$

when all exponents are reduced modulo 2.

If the experiment becomes large, it may be difficult to read the alias summary as shown in Table 21. Therefore, an abbreviated summary (Table 22) is provided to show how much main effect and first-order interaction is aliased with main effects and first- and higher-order interactions. An examination of this table shows that all main effects are aliased only with higher-order interactions. This is an acceptable design for a fractional factorial experiment. Refer to Table 9 to find the specific aliased terms. The particular design acceptability criteria depend upon the problem being studied.

#### DESIGN EVALUATION

Once the alias summary has been generated, the experimenter must determine if the design is acceptable. The acceptability of a design depends upon the aliasing of those effects assumed to be significant by the experimenter. If an insignificant effect is aliased with a significant effect, the design is considered to be a good design. If a few significant effects are aliased, however, the user may define another design or he might use the current design and let the data analysis indicate if an effect that consists of the combination of two potentially significant effects is significant. If a combined effect is significant, the design can be refined to perform the required effect separation (refer to Section 8, Refining Designs).

Once an acceptable design has been generated, the specific observation vectors used to collect data must be found. This collection of observation vectors is called the basic experimental block.

#### BASIC BLOCK DEFINITION

Once the total alias set has been defined, the specific treatment combinations used to collect data must be found. The details of the construction of this block may be skipped by the novice user.

The  $M$  members of the defining contrast set are used to generate the block of treatments and are consequently called generators. If the generators are denoted by

Table 21. Alias Example

EFFECT	ALIASED WITH:		
A	BCE	BD <sup>F</sup>	ACDEF
B	ACE	AD <sup>F</sup>	BCDEF
C	ABE	ABC <sup>D</sup> <sup>F</sup>	DEF
D	ABCDE	AB <sup>F</sup>	CEF
E	ABC	ABDE <sup>F</sup>	CDF
F	ABCE <sup>F</sup>	AB <sup>D</sup>	CDE
AB	CE	D <sup>F</sup>	ABCDEF
AC	BE	BCD <sup>F</sup>	ADEF
AD	BCDE	B <sup>F</sup>	ACEF
AE	BC	BDE <sup>F</sup>	ACDF
AF	BCEF	B <sup>D</sup>	ACDE
BC	AE	ACD <sup>F</sup>	BDEF
BD	ACDE	A <sup>F</sup>	BCEF
BE	AC	ADE <sup>F</sup>	BCDF
BF	ACE <sup>F</sup>	A <sup>D</sup>	BCDE
CD	ABDE	ABC <sup>F</sup>	EF
CE	AB	ABCDEF	DF
CF	ABE <sup>F</sup>	ABCD	DE
DE	ABCD	ABE <sup>F</sup>	CF
DF	ABCDEF	AB	CE
EF	ABCF	ABDE	CD

Table 22. Abbreviated Alias Summary

EFFECT	MAIN	1ST ORDER	HIGHER ORDER
A	0	0	3
B	0	0	3
C	0	0	3
D	0	0	3
E	0	0	3
F	0	0	3
AB	0	2	1
AC	0	1	2
AD	0	1	2
AE	0	1	2
AF	0	1	2
BC	0	1	2
BD	0	1	2
BE	0	1	2
BF	0	1	2
CD	0	1	2
CE	0	2	1
CF	0	1	2
DE	0	1	2
DF	0	2	1
EF	0	1	2



$$G_i = Aaibbic_i \dots i = 1, 2, \dots, M,$$

then the levels in the factorial combination  $x_1 x_2 x_3 \dots$  selected for the block satisfy simultaneously the  $M$  equations

$$a_i x_1 + b_i x_2 + c_i x_3 + \dots = 0 \text{ (modulo } P)$$

$$i = 1, 2, \dots, M$$

Similar equations in which  $1, \dots, (P - 1)$  is used in place of 0 are equally valid; however, the set of treatments defined using the 0, called the Basic Block or the principal block, will be used here.

Consider the one-fourth replicate of an experiment with six factors at two levels, where the defining contrasts are  $I = ABCD = ABDF$ . (The total alias set is  $I = ABCD = ABDF = CDEF$ .) The defining contrasts define two generating equations:

$$G_1 = A1B1C1D0E1F0$$

$$G_2 = A1B1C0D1E0F1$$

The simultaneous equations to be solved are:

$$x_1 + x_2 + x_3 + x_5 = 0 \text{ modulo } 2$$

$$x_1 + x_2 + x_4 + x_6 = 0 \text{ modulo } 2$$

Each of the  $2^6 = 64$  factor combinations is evaluated using this system of equations, and those combinations satisfying the equations form the basic experimental block. Table 23 shows all 26 treatment combinations, and the 16 that form the basic block are shown with an asterisk.

The basic block (or observation vector) for the example is given in Table 24. Note that 0 indicates the factor is at its low level whereas a 1 indicates a factor is at its highest level. Once the basic block is defined, the experimenter must collect the experimental data. Data collection procedures are discussed in the next section.

Table 23. Basic Experimental Block

EQUATION VALUE											
EFFECT	(1)	(2)	(3)	EFFECT	(1)	(2)	(3)	EFFECT	(1)	(2)	(3)
* 000000	0	0	0	010000	1	1	0	* 110000	0	0	0
000001	0	1	1	010001	1	0	1	110001	0	1	1
000010	1	0	1	010010	0	1	1	110010	1	0	1
000011	1	1	0	* 010011	0	0	0	110011	1	1	0
000100	0	1	1	010100	1	0	1	110100	0	1	1
* 000101	0	0	0	010101	1	1	0	* 110101	0	0	0
000110	1	1	0	* 010110	0	0	0	110110	1	1	0
000111	1	0	1	010111	0	1	1	110111	1	0	1
001000	1	0	1	011000	0	1	1	111000	1	0	1
001001	1	1	0	* 011001	0	0	0	111001	1	1	0
* 001010	0	0	0	011010	1	1	0	* 111010	0	0	0
001011	0	1	1	011011	1	0	1	111011	0	1	1
001100	1	1	0	* 011100	0	0	0	111100	1	1	0
001101	1	0	1	011101	0	1	1	111101	1	0	1
001110	0	1	1	011110	1	0	1	111110	0	1	1
* 001111	0	0	0	011111	1	1	0	* 111111	0	0	0

\* IDENTIFIES THOSE TREATMENT COMBINATIONS WHICH WILL BE INCLUDED IN THE BASIC EXPERIMENTAL BLOCK

Table 24. Basic Block Summary

EXPERIMENTAL UNIT	FACTOR LEVEL					
	A	B	C	D	E	F
1	0	0	0	0	0	0
2	0	0	0	1	0	1
3	0	0	1	0	1	0
4	0	0	1	1	1	1
5	0	1	0	0	1	1
6	0	1	0	1	1	0
7	0	1	1	0	0	1
8	0	1	1	1	0	0
9	1	0	0	0	1	1
10	1	0	0	1	1	0
11	1	0	1	0	0	1
12	1	0	1	1	0	0
13	1	1	0	0	0	0
14	1	1	0	1	0	1
15	1	1	1	0	1	0
16	1	1	1	1	1	1

## DATA COLLECTION

Using the observation vectors defined in the basic experimental block, the data collection process is relatively straightforward. The different combinations in the basic block are tested, and the response value is measured.

No consideration has been given to the specific order in which the observation vectors are to be run. The general procedure is to select random combinations until each of the observation vectors in the basic block has been run. This is acceptable unless the experimenter considers a change in the system over time. A system change can be overcome by dividing the basic block into smaller blocks. The assumption is made that the system is relatively homogeneous within each block. Techniques for the construction of blocks are identical to those used to build the alias set. An effect is selected, and the effect equation is generated. For example, consider effect AC. The value of the effect equation for AC equals 0 modulo 2 goes into one block, and the effect equation for AC equals 1 modulo 2 goes into a second block. This causes effect AC to be no longer measurable, i.e., the experimenter cannot know if a response is due to the interaction AC or to a change between blocks.

Blocking procedures are not included in this program. This capability will be added at a later date. The inclusion of this feature, however, requires a trained, experienced user.

Once the data have been collected, they must be analyzed to identify significant effects. This analysis consists of an analysis of variance (ANOVA) or of a regression analysis. Because details concerning ANOVA and regression analysis methods may be found in any statistical analysis text, they have not been included here.

## REFINING DESIGNS

When data have been collected and analyzed from a fractional factorial experiment, the experimenter may determine that he wishes to further examine certain effects or interactions that were confounded in the original design. Finding

a new design in which these effects are separated is called refining the design. This is accomplished by dividing one of the effects to be separated by the other to yield a member of the alias set. This division is defined as a subtraction of corresponding exponents where an exponent is increased by P whenever the subtrahend would have been larger than the minuend. For example, if  $P = 3$ ,

$$\begin{array}{r} 201/102 \quad 201 \quad 104 \\ \quad \quad \underline{-102} \quad \underline{-102} \\ \quad \quad \quad 102 \end{array}$$

The member of the power set that was used to generate this member of the alias set is examined for non-zero columns. Removal of any one of the defining contrasts represented by these columns results in separating the two effects, (i.e., if the member of the power series was found to be 210, removal of either the first or the second defining contrast would accomplish the desired separation).

Removal of a defining contrast results in doubling the total number of trials in a two-level experiment, and tripling the total number of trials in a three-level experiment, although the first portion of the experiment may have already been completed before removal of the defining contrast. This is the price paid for reducing the confounding.

#### IRREGULAR FRACTIONAL FACTORIAL EXPERIMENTS

The generation of fractional experiments uses a  $1/P^N$  design, i.e., a  $1/8$ , a  $1/27$ , or a  $1/64$  design. Although any fraction such as  $k/P^N$  may be constructed, these designs have many problems. Consider the case with five factors at two levels, but with only 24 observation vectors available. One possibility would be a  $3/4$  design based on using a  $1/2$  and  $1/4$  replicate design. If this is used, the total design will have highly correlated estimates that could lead to extremely difficult tests of significance.

Another approach would be to use a  $1/2$  replicate and a  $1/2$  replicate of the unused portion of the larger design. Because of confounding, however, this design provides less information to the experimenter than the  $1/2$  replicate alone. For these reasons, fractional designs other than a  $1/P^N$  are not advantageous.

### SCREENING DESIGNS

One of the common information objectives of human factors engineering research (and indeed of many experiments in general) is to determine the factors that produce a certain result and the relative importance of these factors.

Before a formal experiment can ever be conducted, an investigator is generally required to cull the list of possibly hundreds of potential factors by considering such criteria as:

- Information gained from related research
- Practical constraints of time and money
- Customer interest
- Rational analysis.

Once the least interesting factors have been pared from the list of potentials, fractional factorial designs used as sequential experiments can be of real value.

The term sequential design is generally used to describe experiments in which the yield or response on any unit is known before the experimenter treats the next unit. When an experiment is sequential, the experimenter can stop after every observation or group of observations and examine the results to date before deciding how or whether to continue the experiment.

The term screening design is generally used to describe experiments where an investigator starts with a relatively large number of factors, and by performing a small number of trials, (using a highly fractionated design) can determine that some of the factors are of little significance. These factors can then

be eliminated or screened from further consideration, and another experiment can be designed from the remaining factors. Once the factors that are of importance relative to a given error of prediction have been identified, it may be necessary to perform detailed work on these factors, perhaps using more levels per factor, or creating a mixed level design, or using a central composite design.

## RESPONSE SURFACE DESIGNS

In many investigations, the final goal is to study how the response varies in an experimental region and perhaps to determine the factor combination yielding an optimum response (if an optimum response is suspected to exist). If the  $N$  significant factors are all quantitative with their levels denoted  $X_1, X_2, \dots, X_N$  and the response is quantitative with its level denoted  $Y$ , then  $Y$  is a function  $\emptyset$  of the  $X_1, X_2, \dots, X_N$ ; i.e.,  $Y = \emptyset(X_1, X_2, \dots, X_N)$ . The function  $\emptyset$  is called the response function or response surface. Then the final goal of the investigation is to determine the behavior of the function  $\emptyset$  in the experimental region.

## APPROXIMATIONS OF RESPONSE SURFACES BY POLYNOMIALS

### THE RESPONSE SURFACE

Suppose all  $N$  factors and the response are quantitative and continuous. Denote the levels of the factors by  $X_1, X_2, \dots, X_N$  and denote the level of the response by  $Y$ . Then  $Y$  is a function  $\emptyset$  of  $X_1, X_2, \dots, X_N$  and we can write  $Y = \emptyset(X_1, X_2, \dots, X_N)$ . The function  $\emptyset$  is called the response function. We can also interpret  $Y = \emptyset(X_1, X_2, \dots, X_N)$  as the equation of a surface in  $N+1$  dimensional space, called the response surface. The terms response surface and response function will be used interchangeably.

Note that we have restricted the concept of a response surface to the case where the response and all factors are quantitative. If only some of the factors are quantitative, say the first  $K$ , then one could fix the levels of the qualitative factors, so that they become constants instead of factors. One could then talk about the response function  $Y = \emptyset(X_1, X_2, \dots, X_K)$ . Whether

or not consideration of this restricted response function is worthwhile, and if so, at which levels the qualitative factors should be fixed, must be determined by the investigator.

#### POLYNOMIAL APPROXIMATIONS

In most investigations the nature of the response surface (i.e., the form of the response function) is unknown, so a goal of actually determining the true response function is unattainable. Consequently, we will be satisfied if we can find an approximating function such that the value of the approximating function, in the experimental region under investigation, is sufficiently close to the observed response for our purposes.

The only approximating functions we will consider will be polynomials in the levels of the factors. Polynomials are relatively easy to work with, and in a given region any continuous function can be approximated to any desired degree of accuracy by a polynomial of sufficiently high degree.

For example, suppose an investigation involves three factors. Then some of the possible polynomial approximations are as follows:

1. A linear approximation is a polynomial of degree one and is of the form:

$$Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3$$

2. A quadratic approximation is a polynomial of degree two and is of the form:

$$\begin{aligned} Y = & B_0 + B_{11}X_1^2 + B_{22}X_2^2 + B_{33}X_3^2 \\ & + B_{11}X_1^2 + B_{22}X_2^2 + B_{33}X_3^2 \\ & + B_{12}X_1X_2 + B_{13}X_1X_3 + B_{23}X_2X_3 \end{aligned}$$



3. A cubic approximation is a polynomial of degree three and is of the form:

$$\begin{aligned}
 Y = & B_0 + B_1X_1 + B_2X_2 + B_3X_3 + B_{11}X_1^2 + B_{22}X_2^2 \\
 & + B_{33}X_3^2 + B_{12}X_1X_2 + B_{13}X_1X_3 + B_{23}X_2X_3 \\
 & + B_{111}X_1^3 + B_{222}X_2^3 + B_{333}X_3^3 + B_{112}X_1^2X_2 + B_{122}X_1X_2^2 \\
 & + B_{113}X_1^2X_3 + B_{133}X_1X_3^2 + B_{223}X_2^2X_3 + B_{233}X_2X_3^2 + B_{123}X_1X_2X_3
 \end{aligned}$$

A polynomial of degree greater than three is a straightforward extension of the above cases. However, such polynomials are relatively unlikely to be used for approximating response surfaces.

Note that each of the above polynomials is a complete polynomial in that it contains all possible terms of degree less than or equal to the degree of the polynomial. Also, in the above polynomials each term can be interpreted as a contribution to the response level  $Y$  due to a certain one-factor effect or to a factor interaction. For instance, in the quadratic case

$B_0$  is the base level of the response (the response level when all factors are at level zero)

$B_iX_i$  is the linear effect of the  $i$ -th factor

$B_{ii}X_i^2$  is the quadratic effect of the  $i$ -th factor

$B_{ij}X_iX_j$  is the two-factor interaction effect of the  $i$ -th and  $j$ -th factors.

When we choose the degree of the approximating polynomial, we make certain implicit assumptions about the significance of certain effects. If we choose a linear approximation, we are assuming that all two-factor interactions and

all quadratic one-factor effects are not significant; if we choose a quadratic approximation, we are assuming that all three-factor interactions and all cubic one-factor effects are not significant. If these implicit assumptions are incorrect, the approximating polynomial is likely to fit the observed responses poorly.

The polynomial degree chosen may also vary with the purpose of the approximation and may be different at different phases of a sequential design. For instance, in an investigation whose goal is to determine an optimum response we would normally use linear approximations at each step where we use the method of steepest ascent (see Cochran & Cox, 1957) but then go to a quadratic approximation at the point where the method of steepest ascent finally breaks down.

#### DATA REQUIREMENTS

Having decided what degree of polynomial approximation to use, we must design an experiment that will yield data sufficient to allow a determination of the coefficients of the polynomial. In particular, the following restrictions apply.

1. The number of levels at which the  $i$ -th factor occurs must be at least one greater than the highest power of  $X_i$  in the polynomial.
2. The total number of distinct factor combinations (experimental trials) must be at least as large as the number of polynomial coefficients to be determined.
3. The design of the experiment should not allow any two terms in the polynomial to have their corresponding factor combinations aliased with one another.

Even these three conditions do not guarantee that the experimental data are sufficient to allow determination of all polynomial coefficients. However, an appropriate central composite design (discussed later) will always be sufficient to determine all coefficients of the desired quadratic polynomial.

### USE OF INCOMPLETE POLYNOMIALS

To this point it has been assumed that the approximating polynomial is a complete polynomial, i.e., that it contains all possible terms of degree less than or equal to the degree of the polynomial. This need not be the case. From a complete polynomial we could omit those terms corresponding to effects that are known to be not significant.

For example, suppose there are 6 factors A,B,C,D,E, and F but that it is known that the only significant interactions are the two-factor interactions between A,B, and C. Then instead of the complete quadratic polynomial, which contains 28 terms, one could use the following as an approximating polynomial:

$$Y = B_0 + \sum_{i=1}^6 B_i X_i + \sum_{i=1}^6 B_{ii} X_i^2 \\ + B_{12} X_1 X_2 + B_{13} X_1 X_3 + B_{23} X_2 X_3$$

This incomplete polynomial contains only 16 terms; twelve two-factor interaction terms have been omitted. The possible advantage of an incomplete polynomial is that one may be able to generate sufficient data (to determine the polynomial coefficients) from a smaller experiment than is required for the complete polynomial. The notation  $P^{N-M}$  where P is the number of levels, N is the number of factors, and M is the number of defining contrasts will be used in the following examples to describe the factorial case under consideration. In the above example, a  $2^{6-1}$  factorial (e.g., with I = ABCDEF) would be required to determine all two-factor interactions in the complete polynomial, but a  $2^{6-2}$  factorial (e.g., with I = ABCD = ABEF) would allow determination of all necessary two-factor interactions in the complete polynomial.

### THE CENTRAL COMPOSITE DESIGN

The central composite design is specifically intended to allow determination of a quadratic approximation of the response surface. It is a composite or combination of a full or fractional two-level factorial design and some additional experimental points selected in a particular manner so as to allow a good determination of the quadratic one factor effects. Thus the central

composite design is often appropriate when one suspects that the relationship between the response and the level of a factor is non-linear. In addition, the central composite design should give a good estimate of the response mean and provide an estimate of the precision of the mean.

The central composite design is often especially suitable in a sequential design process. It is sometimes a less costly alternative to three or five-level factorial designs. Some examples of situations where the central composite design might be used are as follows:

1. One has already run a full or fractional factorial experiment and now wants information about possible non-linearity and the shape of the response surface, plus a better estimate of the mean, with a minimum number of additional trials.
2. An investigation involves a small number of continuous quantitative factors and one wants as much information as possible quickly and at low cost.
3. One has already run a full or fractional two-level experiment and wants to expand the experimental region at low cost.

#### DEFINITION OF THE CENTRAL COMPOSITE DESIGN

An N-factor central composite design consists of three components.

1. A full or  $1/2^M$  replicate of a  $2^N$  factorial design, where for each factor the two coded levels are -1 and 1.
2.  $2N$  star or axial points. For each factor there are two corresponding axial points; the given factor has coded level  $-\alpha$  at one point and  $+\alpha$  at the other, whereas all other factors have coded level zero at both points.
3. The center point, where all  $N$  factors have coded level zero.

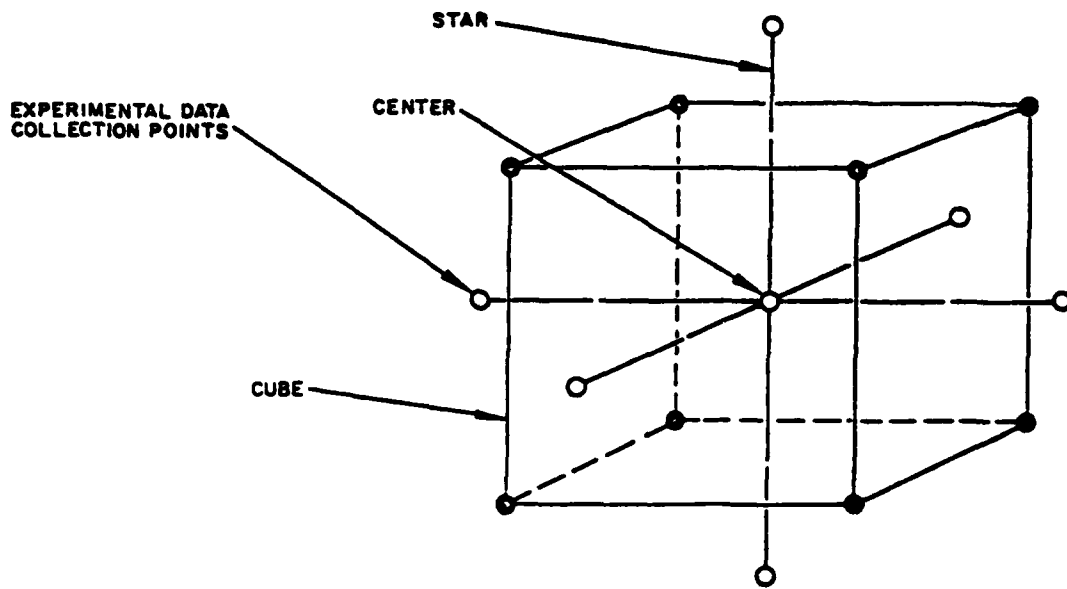


Figure 2. Central Composite Design

For example, a three-factor central composite design would be as follows, where the experimental point with first factor at coded level  $X_1$ , second factor at coded level  $X_2$ , and third factor at coded level  $X_3$  is represented by the vector  $(X_1, X_2, X_3)$ .

1. A full  $2^3$  factorial:  $(-1, -1, -1)$ ,  $(-1, -1, 1)$ ,  $(-1, 1, -1)$ ,  $(-1, 1, 1)$ ,  
 $(1, -1, -1)$ ,  $(1, -1, 1)$ ,  $(1, 1, -1)$ ,  $(1, 1, 1)$
2. 6 axial points:  $(-\alpha, 0, 0)$ ,  $(\alpha, 0, 0)$ ,  $(0, -\alpha, 0)$ ,  $(0, \alpha, 0)$ ,  
 $(0, 0, -\alpha)$ ,  $(0, 0, \alpha)$
3. The center point:  $(0, 0, 0)$

#### CODED VS. REAL WORLD LEVELS

In the central composite design the coded or formal levels of each factor are  $-\alpha$ ,  $-1$ ,  $0$ ,  $1$ , and  $\alpha$ . The real world levels of a factor (i.e., the true quantitative levels of the factor that correspond to the coded levels) must be determined by the researcher. In determining the real world experimental range of levels for a factor, whether for a central composite design or some other design, the following should be considered.

1. If the chosen range is too small, the researcher may erroneously conclude that the factor is not significant (e.g., in a pilot study to determine significant factors), that the one-factor effect of that factor is linear, or that the factor does not interact with other factors.
2. If the chosen range is too large, effects of order higher than the degree of the approximating polynomial may become significant. This could result in a poor fit between the approximating polynomial and the observed data.

Having decided upon the real world experimental range to be used for a particular factor, we can use the following linear transformation to determine the real world level corresponding to any formal level.

$$r = \frac{(r_U - r_L)}{2\alpha} f + \frac{(r_U + r_L)}{2}$$

Here  $r$  is the real world level,  $f$  is the formal or coded level, and  $r_L$  and  $r_U$  are the lower and upper value in the chosen real world range.

If a full or fractional 2 level factorial experiment has already been completed, we can use the following linear transformation to determine the real world level corresponding to any formal level.

$$r = \frac{(r_{+1} - r_{-1})}{2} f + \frac{(r_{+1} + r_{-1})}{2}$$

Here  $r$  is the real world level,  $f$  is the formal or coded level, and  $r_{-1}$  and  $r_{+1}$  are the real world levels of the previously defined 2 level factorial experiment.

In certain instances one might replace the real world levels of a factor by a function of the levels. For instance, if one is using the real world range .001 to .1 but wishes to consider .01 as the midpoint of the range, then one could replace the raw values by their logarithms. This would give  $r_L = \log (.001) = -3$ ,  $r_U = \log (.1) = -1$ , and  $\log (.01) = -2$  is midway between  $r_L$  and  $r_U$ .

The linear transformation would then be:

$$r = -2 + \frac{f}{\alpha}$$

The 5 levels of that factor occurring in the central composite design would be:

coded or formal level	$-\alpha$	-1	0	1	$\alpha$
real world level (log)	-3	$-2-1/\alpha$	-2	$-2+1/\alpha$	-1
real world raw value	.001	$10^{-2-1/\alpha}$	.01	$10^{-2+1/\alpha}$	.1

#### CHOICE OF THE LEVEL $\alpha$

A rotatable design is one that leaves the variance of the estimated response unchanged when the design is rotated about the center. This means that the variance of the estimated response is the same at all points equidistant from the center (0,0, ...,0) of the design. Rotatable designs are desirable in situations where the researcher has no advance knowledge of the response surface or how it is oriented relative to the factor axes and thus has no knowledge of how the variance of the estimated response will vary along the surface.

Box and Hunter (1957) have shown that if the factorial part of the central composite design is a  $2^{N-M}$  factorial in which one-factor effects and two-factor interactions are aliased only with higher order effects, then choosing  $\alpha = 2^{(N-M)/4}$  will make the design rotatable. Although rotatability is not critical, unless we have a reason for doing otherwise we would normally place all  $2N$  axial points at the distance  $\alpha = 2^{(N-M)/4}$  from the center point so that the central composite design is rotatable.

In certain situations, such as in a sequential design process where a  $1/2^M$  fractionation resulted from a prior experiment, there may be one or more factors for which the levels  $\pm 2^{(N-M)/4}$  are not feasible. In such situations we might choose a value of  $\alpha$  different from  $2^{(N-M)/4}$ . Alternatively, the axial points for those factors only could be chosen at a distance other than  $\alpha$  from the center; in fact the two axial points for a factor could be at unequal distances from the center point.

The purpose of each pair of axial points is to increase the precision of the estimate of the corresponding quadratic term  $B_{ij}x_i^2$ . Increasing the distances of these axial points from the center will decrease the variance of this estimated one-factor quadratic effect but will also increase its correlation with other such effects and will increase the danger of bias from higher order effects. On the other hand, if these axial points are within or very near the range from -1 to 1 then they may not significantly increase the precision of the estimate of the quadratic effect  $B_{ij}x_i^2$ .

#### USE OF MULTIPLE OBSERVATIONS AT THE CENTER POINT

Using multiple regression techniques on the experimental data resulting from the central composite design or some other appropriate design, we can derive a polynomial least squares approximation to the response surface. From this process we would also like estimates of the lack of fit and of the experimental error.

1. The lack of fit estimate should indicate the significance of the totality of all effects without corresponding terms in the polynomial. If the approximation is a complete quadratic polynomial these would be the effects represented by terms of degree 3 or greater; e.g.,

$$B_{iii}x_i^3, B_{ijj}x_i^2x_j, B_{ijk}x_i x_j x_k, \text{ etc.}$$

2. The experimental error estimate is just an estimate of the experimental error variance, i.e., of the standard error of the estimated response.

In the central composite design we can provide for an estimate of the experimental error by repeating observations at the center point. If there are  $n_0$  replications of the center point, they provide  $n_0 - 1$  degrees of freedom for estimating the experimental error. Box and Hunter (1957) determined values of  $n_0$  for which the variance of the estimated response is approximately the same at the center and at unity distance from the center of a rotatable central composite design. In this case the standard error is roughly the same at all points within the sphere of radius one and can be approximated by the standard error at the center point; the graph in Box and Hunter indicates that the



standard error increases fairly rapidly outside the sphere of radius one. From Box and Hunter's results it follows that the formula for such a value of  $n_0$  is

$$n_0 = (2^{N-M} + 4 + 4.2^{(N-M/2)}) \left( \frac{N + 3 + 9N^2 + 14N - 7}{4N + 8} \right) - 2^{N-M-2N}$$

Table 25 gives such values of  $n_0$  (rounded to the nearest integer) for rotatable central composite designs.

Table 25. Replicates of Center Point for Nearly Uniform Variance.

N	M = 0	M = 1	M = 2	M = 3
2	5			
3	6			
4	7			
5	10	6		
6	15	9		
7	21	14		
8	28	20	13	
9	37	25	16	
10	43	38	29	18

After deriving an approximate polynomial, one can do an analysis of variance. By partitioning the total sum of squares into a number of parts, each the contribution due to a source of interest, one can get a lack of fit estimate. For instance, for a quadratic approximation resulting from a central composite design one could partition the sum of squares into four parts:

- (a) sum of squares due to first order terms
- (b) sum of squares due to second order terms
- (c) sum of squares due to experimental error (from the  $n_0$  replications of the center point)
- (d) sum of squares due to lack of fit.

The sum of squares in (d) is found by subtracting from the total sum of squares the sum of squares in (a), (b) and (c). By comparing the mean squares for the various parts in the partition, one can determine the relative significance of each part of the partition.

#### RESTRICTIONS ON A CENTRAL COMPOSITE DESIGN

The following comments are appropriate to a discussion of the restrictions which must be placed on a central composite design in order to allow determination of a quadratic approximation of a response surface.

1. In that part of the central composite design consisting only of the axial and center points, each factor must appear at three separate levels  $-\alpha$ , 0, and  $\alpha$ , with the levels of all other factors fixed at 0. Thus the basic level  $B_0$ , the linear one-factor terms  $B_i X_i$ , and the quadratic one-factor terms  $B_{ii} X_i^2$  could be estimated from these points alone.
2. The two-factor interaction terms  $B_{ij} X_i X_j$  must be estimated from the factorial part of the central composite design, since in the center and axial points portion of the central composite design only one factor at a time is being varied.

From these two comments one can see that when using a central composite design to fit a quadratic polynomial to a response surface, the basic restriction on the central composite design is imposed by the requirement that the factorial part of the central composite design allow estimation of all two-factor interaction terms in the polynomial. In particular, when fitting a complete quadratic polynomial this requirement is that no two-factor effect be aliased with I or any other two-factor effect. In practice, we strengthen this requirement to "no one-factor effect or two-factor interaction can be aliased with I, a one-factor effect, or a two-factor interaction."

When fitting an incomplete quadratic polynomial this restriction can be relaxed. Then we usually require that each of the main effects and two-factor interaction effects with a corresponding term in the polynomial not be aliased with another of these effects.

To have one-factor effects and two-factor interactions aliased only with higher order effects it is necessary and sufficient that the non-I terms of the total alias set all involve at least five factors. For a given number of factors, Table 26 gives the smallest possible fractional factorial with this property.

Table 26. Minimum Fractionation Confounding Only Higher Order Effects

NUMBER N OF FACTORS	MINIMUM FRACTIONATION	NUMBER OF OBSERVATION VECTORS IN THE FRACTIONATION
2	FULL	4
3	FULL	8
4	FULL	16
5	1/2	16
6	1/2	32
7	1/2	64
8	1/4	64
9	1/4	128
10	1/8	128
11	1/16	128
12	1/16	256
13	1/32	256
14	1/64	256
15	1/128	256
16	1/128	512
17	1/256	512

As can be seen from the above table, when six or more factors are involved the central composite design requires a fairly large number of factor combinations even for a minimum fractional factorial. Thus, when feasible, it may be worthwhile to use an incomplete quadratic polynomial. For instance, if the only significant interactions are two-factor interactions among just four of the factors, then a 16 unit fractional factorial will suffice for 6, 7, or 8 factors and a 32 unit fractional factorial will suffice for 9 or 10 factors. Predefined Designs 2.6.16, 2.7.16, 2.8.16, and 2.9.32 (see section 15, Predefined Designs) are examples of such fractional factorials if the only significant interactions are taken to be two-factor interactions among A,C,D, and E.

### COMPARISON WITH THE THREE-LEVEL FACTORIAL

The central composite design is specifically designed to allow a quadratic approximation of a response surface. Other designs can also be used for this purpose. The most obvious alternate design is the three-level factorial. For example, if we want a complete quadratic approximation of a five-factor response surface, then we could either use a central composite design involving a  $2^{5-1}$  factorial or use a  $3^{5-1}$  factorial. The central composite design will often have a number of advantages over the three level factorial.

1. The central composite design is often more economical than alternate designs. In the above example the central composite design requires only 27 points whereas the  $3^{5-1}$  design requires 81 points, where neither count includes multiple replicates of the center point.
2. The central composite design is often appropriate in a sequential design process. If one has already run a  $2^{N-M}$  factorial experiment from which one has determined the desirability of finding a quadratic approximation of the response surface, then it may be sufficient to collect additional observations on the axial and central points to complete the central composite design.
3. In the central composite design every factor appears at five levels whereas in the  $3^N$  factorial every factor appears at only three levels. Thus the central composite design should give better relative precision of the one-factor quadratic terms  $B_{ij}X_i^2$ .

If it turns out that a quadratic approximation yields a poor fit, then the 3-level factorial may be more advantageous. A complete  $3^N$  factorial allows determination of third degree terms with each factor exponent  $\leq 2$ , whereas the central composite design involving a complete  $2^N$  factorial allows determination of third and fourth degree one-factor terms and third degree interaction terms with all factor exponents  $\leq 1$ . Thus if we want to go to a cubic approximation without additional experimental points, whether or not the terms  $B_{ijk}X_i^3$  are more significant than the terms  $B_{ijj}X_i^2X_j$  determines whether or not the central composite design allows a better fit than the  $3^N$  factorial. In

any case, as  $N$  increases, the  $3^N$  factorial becomes much more expensive than the central composite design.

#### DATA ANALYSIS

Once the experimental data have been collected according to the central composite design, the data must be analyzed. The first step in the analysis process is to use multiple regression techniques to derive the polynomial (of the chosen form) which provides a least squares best fit to the experimental data; that is, the polynomial which minimizes the sum of the squares of the differences between the observed response level and the response level predicted by the polynomial. The details of this derivation are documented in many of the references and in most statistics texts and are not presented here. From this derivation process we can also get estimates of  $\sigma^2$  (the variance of the estimated response) and of the variances and covariances of the estimates of the coefficients of the derived polynomial.

The second step in the analysis process is an analysis of variance. Here we tabulate the degrees of freedom, the sum of squares, and the mean square for each part of the partition of sum of squares. A comparison of the mean squares indicates the relative significance of each part. In particular, a mean square corresponding to lack of fit that is substantially larger than the mean square corresponding to experimental error indicates that the derived polynomial does not fit the true response surface very well.

If one is satisfied that the derived polynomial provides an adequate fit, then there are several additional analysis steps one might consider.

1. One could try to find a stationary point of the quadratic surface; i.e., a point where the partial derivatives of the polynomial with respect to the factor levels are all zero.
2. If one does find a stationary point, one could perform a translation of axes and a rotation of axes to transform the quadratic function to its canonical form  $Y = \gamma_0 + \gamma_1 Z_1^2 + \gamma_2 Z_2^2 + \dots + \gamma_N Z_N^2$ . In this canonical form the origin is at the stationary point and the principal

axes of the quadratic surface are also coordinate axes, so it is relatively easy to determine the shape of the surface.

3. One can determine whether the stationary point is a maximum point or a saddle point.
4. If the stationary point is a saddle point, one can determine the directions of most promise for a further search for an optimum point.

These steps are appropriate mainly in those situations where one might suspect the existence of an optimum response. In particular these steps depend on the existence of a stationary point, which in many situations may not exist.

In any analysis of the derived polynomial, it should be kept in mind that even if the polynomial provides a good fit the polynomial should be considered to be a good approximation only within the region bounded by the experimental ranges of the factors. If one extrapolates beyond that region then any conclusions drawn may be suspect. For instance, if the analysis of the quadratic function indicates a stationary point is a maximum point, but the stationary point is outside the experimental region, then the stationary point may not be very close to a real maximum. In this situation it might be best to do further investigation by moving toward this calculated maximum point and running additional observations.

## MIXED LEVEL DESIGNS

### COMBINED QUALITATIVE AND QUANTITATIVE FACTORS

The simplest form of combined level design involves an experiment which includes both qualitative and quantitative factors with all factors involving the same number of levels. As an example, consider an experiment involving five factors. Factors A, B, and C are quantitative and D and E are qualitative. A two level design is to be used. This may be expressed as a  $2^3 2^2$  design. Fractionation should not be applied to qualitative variables, so only the  $2^3$  portion of this design can be fractionated. Possible fractionations for this design are as shown in Table 27.

Table 27. 5 Factor, 2 Level, Mixed Qualitative and Quantitative Designs

FRACTIONATION	NO. OF TRIALS	DEFINING CONTRASTS	ALIAS SET
NONE	32	NONE	NONE
1/2	16	I = ABC	ABC
1/4	8	I = AB, I = BC	AB,BC,AC

As another example, consider the same five factor problem in which a three level design is to be used. This is shown in Table 28.

Table 28. 5 Factor, 3 Level, Mixed Qualitative and Quantitative Designs

FRACTIONATION	NO. OF TRIALS	DEFINING CONTRASTS	ALIAS SET
NONE	243	NONE	NONE
1/3	81	I = ABC	ABC,A2B2C2
1/9	27	I = AB, I = BC	AB,A2B2,BC,B2C2 AC,AC2,A2C,A2BC2

Thus any grouping of factors at a given level which consists of both qualitative and quantitative factors should be expressed as

$PQR$  where  $P$  = no of levels

$Q$  = no of quantitative factors

$R$  = no of qualitative factors

Fractionation should only be applied to the  $PQ$  portion of this expression.

### 2K3L DESIGNS

If an experiment involves only quantitative factors, some at one set of levels (two levels for example) and the other at a different number of levels (e.g., three levels) it requires a mixed level design. A mixed level design of this type is most simply approached by considering each portion of the design separately, assigning defining contrasts for each portion of the design, and performing an optimum fractionation for each portion of the design followed by a cross multiplication of the resultants for each portion of the design.

As an example, a  $2^K 3^L$  design could be defined for a  $1/2$  fraction by defining the basic block for a  $1/2$  fraction of the  $2^K$  which would be assembled with the complete  $3^L$  to arrive at the design.

If  $K = 3$  and  $L = 2$ , the resultant component would be as follows:

<u>2 Level</u>	<u>3 Level</u>	<u>Design</u>			
000	00	00000	01100	10100	11000
011	01	00001	01101	10101	11001
101	02	00002	01102	10102	11002
110	10	00010	01110	10110	11010
	11	00011	01111	10111	11011
	12	00012	01112	10112	11012
	20	00020	01120	10120	11020
	21	00021	01121	10121	11021
	22	00022	01122	10122	11022

Upon examining the resulting design, we find that the levels are not "mixed" as well as they might be. As an example, when the first two (2) level factors are at the same level (either both low or both high), although the three level term is exercised at all levels, the third 2 level term is always at its low level.

An approach to improving the "mixing" of levels in the design is to perform a fractionation upon all portions of the design and assemble the resulting fractions to arrive at the desired fraction. In our sample case this would be done as follows:

If a  $1/2$  fraction is desired, the two level portion is divided into two parts  $S_1$  and  $S_2$ .  $S_1$  is the basic block and  $S_2$  is the remaining block. The three level term is divided into  $S'_1$ ,  $S'_2$ , and  $S'_3$  where  $S'_1 = 0 \pmod{3}$ ,  $S'_2 = 1 \pmod{3}$  and  $S'_3 = 2 \pmod{3}$ . The treatment combinations are then:



<u>2 Level</u>		<u>3 Level</u>		
<u>S<sub>1</sub></u>	<u>S<sub>2</sub></u>	<u>S<sub>1</sub>'</u>	<u>S<sub>2</sub>'</u>	<u>S<sub>3</sub>'</u>
000	001	00	01	02
011	010	12	10	11
101	100	21	22	20
110	111			

The experimental plan can then be formulated as:

$$S_1 S_1' + S_2 S_3' + S_1 S_3' \text{ or fractionally } \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}$$

The resulting design is now:

00000	00101	00002
01100	01001	01102
10100	10001	10102
11000	11101	11002
00012	00110	00011
01112	01010	01111
10112	10010	10111
11012	11110	11011
00021	00122	00020
01121	01022	01120
10121	10022	10120
11021	11122	11020

This design results in a better mixing than the original design. A symmetrical design, in which each fraction for a portion of the design is used the same number of times as every other fraction for that portion of the experimental design, would provide the optimum level of mixing. However, this cannot be achieved except in a few cases.

The first step in the process of formulating a mixed level design is to determine the number of components in the experimental plan. This will be illustrated for the  $2^K 3^L$  design,  $1/2$  fractionation.

The experimental plan will consist of:

$$1/2 = G \frac{1}{2^M} \cdot \frac{1}{3^P} \text{ where } G = 3^P 2^{M-1} \\ \text{and } P \leq L \\ M \leq K$$

If  $K = 3$ ,  $L = 2$  ( $2^3 3^2$ ), the possibilities are:

$$1/2 = 3 \frac{1}{2} \cdot \frac{1}{3} = 6 \frac{1}{4} \cdot \frac{1}{3} = 12 \frac{1}{8} \cdot \frac{1}{3}$$

Experimental plans corresponding to these are:

$$S_1 S_1' + S_2 S_2' + S_3 S_3'$$

or  $S_1 S_1' + S_2 S_2' + S_3 S_3' + S_4 S_1' + S_2 S_3'$

or  $S_1 S_1' + S_2 S_2' + S_3 S_3' + S_4 S_1' + S_5 S_2' + S_6 S_3' + S_7 S_1' + S_8 S_2'$

$$+ S_1 S_3' + S_2 S_1' + S_3 S_2' + S_4 S_3'$$

For simplicity, the 3 element plan would be selected since the more complex plans do not offer significant advantages.

#### PREDEFINED DESIGNS

An experimenter frequently requires a design for which he is unable to find the appropriate aliasing. This section includes a set of predefined designs that allow for all main effects to be measurable. Main effects are aliased with only high-order interactions. Also, most two-factor interactions are measurable.

To aid the user, the different experimental designs are identified by the notation L.F.S., where L is the number of levels per factor, 2 or 3; F is the number of factors; and S is the number of observation vectors in the fractional design (e.g., 2.4.8 = 2 levels, 4 factors, and 8 units). The sources of these predefined designs are C&C (Cochran and Cox, 1957) NBS #48 and NBS #54. Some predefined designs were developed as a result of this study.

Design 2.4.8

$2^4$  factorial in 8 units  
1/2 replicate  
I = ABCD  
C&C, p. 276

Design 2.5.8

$2^5$  factorial in 8 units  
1/4 replicate  
I = ABE = CDE  
C&C, p. 277

Design 2.5.16

$2^5$  factorial in 16 units  
1/2 replicate  
I = ABCDE  
C&C, p. 277

Design 2.7.8

$2^7$  factorial in 8 units  
1/16 replicate  
I = ABG = ACE = ADF = BCF  
C&C, p. 280

Design 2.7.26

$2^7$  factorial in 16 units  
1/8 replicate  
I = ABCD = ABEF = ACEG  
C&C, p. 280

Design 2.6.8

$2^6$  factorial in 8 units  
1/8 replicate  
I = ACE = ADF = BCF  
C&C, p. 278

Design 2.6.16

$2^6$  factorial in 16 units  
1/4 replicate  
I = ABCE = ABDF  
C&C, p. 278

Design 2.6.32

$2^6$  factorial in 32 units  
1/2 replicate  
I = ABCDEF  
C&C, p. 279

Design 2.8.32

$2^8$  factorial in 32 units  
1/8 replicate  
I = BCDH = BDFG = ABCEH  
C&C, p. 286

Design 2.8.64

$2^8$  factorial in 64 units  
1/4 replicate  
I = ABCEG = ABDFH  
C&C, p. 287

Design 2.7.32

$2^7$  factorial in 32 units

1/4 replicate

I = ABCDE = ABCFG

C&C, p. 281

Design 2.7.64

$2^7$  factorial in 64 units

1/2 replicate

I = ABCDEFG

C&C, p. 283

Design 2.8.16

$2^8$  factorial in 16 units

1/16 replicate

I = ABCD = ABEF = ABGH = ACEH

C&C, p. 285

Design 2.9.128

$2^9$  factorial in 128 units

1/4 replicate

I = ABCEGJ = ABDFHJ

NBS #48, p. 24

Design 2.9.256

$2^9$  factorial in 256 units

1/2 replicate

I = ABDEFGHJ

NBS #48, p. 17

Design 2.10.64

$2^{10}$  factorial in 64 units

1/16 replicate

I = ABCDJK = ABEFJ = BCEGJK = ABCDEFGH

NBS #48, p. 44

Design 2.8.128

$2^8$  factorial in 128 units

1/2 replicate

I = ABCDEFGH

C&C, p. 288

Design 2.9.32

$2^9$  factorial in 32 units

1/16 replicate

I = ABCD = ABEF = BCEG = EFGHJ

NBS #48, p. 43

Design 2.9.64

$2^9$  factorial in 64 units

1/8 replicate

I = ABEGHJ = ACFGJ = ABCD

NBS #48, p. 33

Design 2.10.512

$2^{10}$  factorial in 512 units

1/2 replicate

I = ABCDEFGHJK

Design 3.3.9

$3^3$  factorial in 9 units

1/3 replicate

I = ABC

Developed Design

Design 3.4.9

$3^4$  factorial in 9 units

1/9 replicate

I = ABC = BC<sup>2</sup>D

Developed Design

Design 2.10.128

$2^{10}$  factorial in 128 units

1/8 replicate

$I = ABEGHJ = ACFGJK = ABCDK$

NBS #48, p. 36

Design 2.10.256

$2^{10}$  factorial in 256 units

1/4 replicate

$I = ABCDEFG = ABCDHJK$

NBS #48, p. 29

Design 3.5.81

$3^5$  factorial in 81 units

1/3 replicate

$I = ABCDE$

NBS #54, p. 11

Design 3.6.27

$3^6$  factorial in 27 units

1/27 replicate

$I = ABCDEF^2 = BC^2EF^2 = ABCE$

Developed Design

Design 3.6.81

$3^6$  factorial in 81 units

1/9 replicate

$I = ACDE = BC^2DE^2F$

NBS #54, p. 19

Design 3.4.27

$3^4$  factorial in 27 units

1/3 replicate

$I = ABCD$

NBS #54, p. 11

Design 3.5.27

$3^5$  factorial in 27 units

1/9 replicate

$I = ABCDE = ABC^2$

Developed Design

Design 3.7.243

$3^7$  factorial in 243 units

1/9 replicate

$I = ABCDE = CD^2EF^2G^2$

NBS #54, p. 20

Design 3.7.729

$3^7$  factorial in 729 units

1/3 replicate

$I = AB^2CDE^2FG$

NBS #54, p. 17

Design 3.8.81

$3^8$  factorial in 81 units

1/81 replicate

$I = BCDEFG = ACDE^2F^2H = AC^2D^2FG =$

$BC^2F^2C$

Developed Design

Design 3.6.243

36 factorial in 243 units

1/3 replicate

$$I = AB^2CDE^2F$$

NBS #54, p. 14

Design 3.7.27

37 factorial in 27 units

1/81 replicate

$$I = ACDEF^2 = BC^2EF^2G =$$

$$ABC E G^2 = AB^2CD^2E^2F^2G^2$$

Developed Design

Design 3.7.81

37 factorial in 81 units

1/27 replicate

$$I = ACDEF^2G = BC^2EF^2G = ABCEG^2$$

NBS #54, p. 23

Design 3.9.243

39 factorial in 243 units

1/81 replicate

$$I = BCDEFG = ACDE^2F^2H =$$

$$ABD^2E^2FJ = ABC^2EF^2$$

NBS #54, p. 31

Design 3.10.243

310 factorial in 243 units

1/243 replicate

$$I = BCDEFG = ACDE^2F^2H =$$

$$ABD^2E^2FJ = ABC^2EF^2 =$$

$$AB^2C^2DFK$$

Developed Design

Design 3.8.243

38 factorial in 243 units

1/27 replicate

$$I = BCDEFG = ACDE^2F^2H = ABD^2E^2F$$

NBS #54, p. 25

Design 3.8.729

38 factorial in 729 units

1/9 replicate

$$I = ABCDEH^2 = CD^2EF^2G^2$$

NBS #54, p. 23

Design 3.9.81

39 factorial in 81 units

1/243 replicate

$$I = BCDEFG = ACDE^2F^2H = ABD^2E^2FJ =$$

$$AB^2C^2DF = ABC^2EF^2$$

NBS #54, p. 36

Design 3.9.729

39 factorial in 729 units

1/27 replicate

$$I = BCDEFG = ACDE^2F^2H = ABD^2E^2FJ$$

NBS #54, p. 26

## USER'S APPENDIX GUIDE WITH EXAMPLES

### INTRODUCTION

This appendix describes the procedures for using the AED computer program. The "conversational mode" of the program operation is illustrated. The functions of each of the program segments are explained, and a listing is included of the program input and output for an example from each segment.

### SYSTEM STRUCTURE

The automated experimental design program is divided into five program segments. Upon execution of the program, a display menu is presented. The user may select one of the five by inputting the number of the segment he wishes. The program echoes the input and requests the user to input an identification label to identify the run. The I.D. label may be any alphanumeric string.

The menu, entry request, and I.D. label request occur whenever a segment is completed and the program is ready to enter another segment. The actual test displayed and a system "walk-through" will be included with the explanation of the use of each segment in the following sections.

### BASIC TERMINOLOGY

Segment 1--Basic Terminology--provides the user with a basic introduction to the process of experimental design. A description of the program assumptions, vocabulary, and a discussion of the rationale behind experimental designs are provided. This material is essentially a tutorial for the user who is unfamiliar with experimental design. Once the user is acquainted with the design process, there should be no need to enter this segment except for an occasional review session. User input is explained in this segment.

BASIC TERMINOLOGY



WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC TERMINOLOGY

PLEASE ENTER THIS RUN PROBLEM I.D.  
BASIC TERMINOLOGY TEST RUN

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.  
THIS PROGRAM PROVIDES THE OPTION FOR FORMULATING SCREENING  
DESIGNS OR RESPONSE SURFACE DESIGNS.

---SCREENING DESIGNS---

SCREENING DESIGNS ASSUME THAT YOUR OBJECTIVE IS TO  
DETERMINE FACTORS THAT PRODUCE A CERTAIN RESULT AND THE  
RELATIVE IMPORTANCE OF THESE FACTORS. IN THIS CASE YOU  
HAVE ALREADY OBTAINED BY OBSERVATION AND INTUITION AN IDEA  
OF THOSE FACTORS THAT MAY POSSIBLY INFLUENCE THE OUTCOME  
OF THE EXPERIMENT. IN THIS CASE, FRACTIONAL DESIGNS ARE OF  
REAL VALUE. THEY MAY BE USED TO DETERMINE WHICH OF THE  
POSSIBLE FACTORS ARE OF IMPORTANCE RELATIVE TO A GIVEN ERROR  
OF PREDICTION. ONCE THESE FACTORS HAVE BEEN DISCOVERED, IT  
IS NECESSARY TO PERFORM DETAILED WORK ON THE FACTORS, POSSIBLY

EVEN ONE AT A TIME IN ORDER TO FORMULATE A LAW RELATING RESPONSE TO THE  
LEVEL FOR EACH FACTOR.

HIT RETURN WHEN READY TO CONTINUE.

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

SCREENING DESIGNS -- A CLASS OF FRACTIONAL FACTORIALS --  
ARE SYSTEMATIC DATA COLLECTION PLANS THAT ENABLE THE EFFECTS  
OF A VERY LARGE NUMBER OF FACTORS TO BE ESTIMATED ECONOMICALLY.  
SCREENING DESIGNS ARE USED PRIMARILY IN THE SECOND PHASE OF  
A TOTAL RESEARCH PROGRAM WHERE THEY ARE INTENDED TO DETERMINE  
WHICH OF THE GREAT MANY FACTORS HAVE NON-TRIVIAL EFFECTS ON  
THE PERFORMANCE OF A PARTICULAR TASK. SCREENING DESIGNS ARE  
TO BE USED TO IDENTIFY IMPORTANT FACTORS, NOT TO OBTAIN AN ACCURATE  
REPRESENTATION OF THE EXPERIMENTAL SPACE. THIS LATTER OPERATION WILL  
OCCUR IN SUBSEQUENT PHASES OF THE RESEARCH PROGRAM.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

THE SCREENING DESIGNS PROVIDE A MEANS OF EXAMINING A GREAT NUMBER  
OF FACTORS WITH THE MAXIMUM AMOUNT OF INFORMATION WITH A MINIMUM  
AMOUNT OF REDUNDANCE AND RELATIVELY FEW TRIALS. WHAT THE RESULTS  
FROM MANY LITTLE TRADITIONAL EXPERIMENTS CANNOT DO, BUT WHICH RESULTS  
FROM THE SCREENING DESIGN CAN DO IS TO ORDER THE FACTORS ACCORDING TO  
SIZE OF THEIR EFFECTS AND TO DISCOVER INTERACTIONS AMONG FACTORS THAT  
APPEAR WITHIN THE SAME EXPERIMENT. SCREENING DESIGNS DO ALL THIS  
ECONOMICALLY FOR THEY CAN BE USED TO STUDY FACTORS WITH A SMALL NUMBER  
OF TRIALS (ALTHOUGH THE SIZE OF THE DESIGNS IN THIS PROGRAM WILL ALL BE  
EQUAL TO A POWER OF 2 OR 3 OR 5). THE EFFECTS OBTAINED FROM SCREENING STUDIES  
NOT ONLY PERMIT THE RANKING OF FACTOR EFFECTS ON A QUANTITATIVE SCALE,  
BUT CAN PROVIDE AN EQUATION APPROXIMATING THE EXPERIMENTAL SPACE IF  
THAT SPACE CAN BE REPRESENTED BY A LINEAR MODEL.

HIT RETURN TO CONTINUE

THE BEAUTY OF USING A SCREENING DESIGN IS THAT ONCE THE IMPORTANT FACTORS HAVE BEEN IDENTIFIED (STEP ONE), THE SAME DATA CAN BE USED, IF SUPPLEMENTED BY RELATIVELY FEW ADDITIONAL TRIALS AT NEW EXPERIMENTAL CONDITIONS, TO COMPLETE A RESPONSE SURFACE (STEP TWO) CAPABLE OF ACCURATELY APPROXIMATING THE EXPERIMENTAL SPACE DEFINED BY THE ORIGINAL SET OF FACTORS.

IN USING A SCREENING DESIGN, THE EXPERIMENTER MUST ASK CERTAIN QUESTIONS ABOUT EACH FACTOR.

DOES THE FACTOR HAVE AN EFFECT ON PERFORMANCE? DOES THE FACTOR ACCOUNT FOR A MEANINGFUL PROPORTION OF THE VARIANCE IN THE EXPERIMENT? DOES INCLUDING THE FACTOR MATERIALLY IMPROVE THE ABILITY TO PREDICT PERFORMANCE UNDER OPERATIONAL CONDITIONS? COULD THE OBSERVED EFFECT HAVE BEEN DUE TO CHANCE? CAN THE CUMULATIVE EFFECTS OF A LARGE NUMBER OF NON-CRITICAL FACTORS BE IGNORED? DO YOU WANT TO REVIEW THIS DISCUSSION OF SCREENING DESIGNS?

NO

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

---RESPONSE SURFACE DESIGNS---

THE FINAL GOAL IN MANY INVESTIGATIONS IS TO DETERMINE THE RESPONSE SURFACE IN AN EXPERIMENTAL REGION. USUALLY THE NATURE OF THE RESPONSE SURFACE (I.E., THE FORM OF THE RESPONSE FUNCTION) IS UNKNOWN. SINCE ACTUALLY DETERMINING THE TRUE RESPONSE FUNCTION IS UNATTAINABLE, AN APPROXIMATING FUNCTION MUST BE USED. POLYNOMIALS IN THE LEVELS OF THE FACTORS ARE SUITABLE APPROXIMATING FUNCTIONS. POLYNOMIALS ARE RELATIVELY EASY TO WORK WITH, AND IN A GIVEN REGION ANY CONTINUOUS FUNCTION CAN BE APPROXIMATED TO ANY DESIRED LEVEL OF ACCURACY BY A POLYNOMIAL OF SUFFICIENTLY HIGH DEGREE. BASIC FACTORIAL DESIGNS AND MIXED LEVEL DESIGNS CAN PROVIDE EXPERIMENTAL DATA SUFFICIENT TO DETERMINE THE POLYNOMIAL COEFFICIENTS, AND APPROPRIATE CENTRAL COMPOSITE DESIGNS WILL ALWAYS BE SUFFICIENT TO DETERMINE ALL THE COEFFICIENTS OF A QUADRATIC POLYNOMIAL.

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

---CENTRAL COMPOSITE DESIGNS---

THE CENTRAL COMPOSITE DESIGN IS AN EXPERIMENTAL DESIGN INTENDED TO ALLOW A QUADRATIC POLYNOMIAL APPROXIMATION OF A RESPONSE FUNCTION WHEN ALL FACTORS AND THE RESPONSE ARE QUANTITATIVE. THE CENTRAL COMPOSITE DESIGN CONSISTS OF A FULL OR FRACTIONAL TWO-LEVEL FACTORIAL DESIGN PLUS ADDITIONAL AXIAL POINTS AND A CENTER POINT NECESSARY

TO APPROXIMATE THE RESPONSE SURFACE.

CENTRAL COMPOSITE DESIGNS REDUCE THE SIZE OF THE EXPERIMENT BY ELIMINATING DATA COLLECTION IN THOSE PARTS OF THE EXPERIMENTAL REGION WHICH ARE LEAST INTERESTING. AN EXPERIMENTER WILL NORMALLY KNOW ENOUGH ABOUT THE PROBLEM TO LOCALIZE THE EXPERIMENT WITHIN THE REGION OF GREATEST INTEREST.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

IF THE FACTORIAL PART OF A CENTRAL COMPOSITE DESIGN HAS ONE-FACTOR EFFECTS AND TWO-FACTOR INTERACTIONS ALIASED ONLY WITH HIGHER ORDER EFFECTS, THEN CHOOSING A SPECIFIC VALUE FOR THE CODED LEVEL OF ALPHA WILL CAUSE THE DESIGN TO BE ROTATABLE. FOR ROTATABLE CENTRAL COMPOSITE DESIGNS A FORMULA IS GIVEN FOR THE NUMBER OF REPLICATIONS OF THE CENTER POINT REQUIRED TO CAUSE THE VARIANCE OF THE ESTIMATED RESPONSE TO BE APPROXIMATELY CONSTANT THROUGHOUT THE SPHERE OF RADIUS ONE.

THE CENTRAL COMPOSITE DESIGN CAN PROVIDE FOR AN ESTIMATE OF THE EXPERIMENTAL ERROR AND FOR AN ESTIMATE OF HOW WELL THE DERIVED QUADRATIC APPROXIMATION FITS THE RESPONSE SURFACE.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

THE CENTRAL COMPOSITE DESIGN CAN BE USED IN THE FOLLOWING SITUATIONS.

1. THE EXPERIMENTER HAS ALREADY RUN A TWO-LEVEL FACTORIAL EXPERIMENT AND NOW WANTS INFORMATION ABOUT POSSIBLE NON-LINEARITY AND THE SHAPE OF THE RESPONSE SURFACE.

2. ONLY A SMALL NUMBER OF FACTORS IS INVOLVED AND THE EXPERIMENTER WANTS AS MUCH INFORMATION ABOUT THE BEHAVIOR OF THE RESPONSE AS POSSIBLE, QUICKLY AND AT LOW COST.

3. THE EXPERIMENTER HAS ALREADY RUN A TWO-LEVEL FACTORIAL EXPERIMENT AND NOW WANT TO EXPAND THE EXPERIMENTAL REGION AS CHEAPLY AS POSSIBLE.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

THE CENTRAL COMPOSITE DESIGN SHOULD NOT BE USED IN THE FOLLOWING SITUATIONS.

1. ALL THAT IS DESIRED IS A SCREEN FOR SIGNIFICANT FACTORS.

2. SOME FACTORS ARE QUALITATIVE.

3. INTERACTION EFFECTS ARE NON-LINEAR.

DO YOU WANT TO REVIEW THIS DISCUSSION OF RESPONSE SURFACE DESIGNS?

NO

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

YOU WILL BE ASKED A SERIES OF QUESTIONS.

SOME OF THESE REQUIRE ONLY A YES OR NO RESPONSE. YOU MAY ENTER A Y OR YES OR N OR NO TO THESE QUESTIONS.

DO YOU WANT TO CONTINUE WITH THESE BASIC INSTRUCTIONS?

YES

The user may review these basic instructions A "NO" response would skip the remainder of this segment.

BASIC TERMINOLOGY TEST RUN  
\*\*\*\*\*

THESE INSTRUCTIONS CONSIST OF A DEFINITION OF YOUR RESPONSES FOR NUMBERS AND AN EXPLANATION OF THE VARIOUS TERMS USED IN THE DESIGN OF EXPERIMENTS.

WHEN A NUMBER IS REQUESTED FOR INPUT, YOU MUST ENTER THE VALUE AS +1 or -2 or '96, ETC., OR 11.67 or -3.26.  
THE NUMBER CANNOT BE INPUT AS A FRACTION (1/2).  
I WILL ECHO EACH VALUE INPUT AND GIVE YOU THE OPPORTUNITY TO CORRECT IT IF NECESSARY.

INPUT A NUMBER OF YOUR OWN CHOICE AS A TEST  
112.345  
IS THIS INPUT CORRECT: 112.345  
NO

INPUT A NUMBER OF YOUR OWN CHOICE AS A TEST  
99.76  
IS THIS INPUT CORRECT: 99.76  
YES

WE WILL NOW DEFINE THE VARIOUS TERMS USED IN THE DESIGN OF EXPERIMENTS PROGRAM.  
DO YOU WANT TO REVIEW THIS INTRODUCTION?  
NO

Data input is critical to the use of this program. Data are echoed and the user may change/correct his response. Note that the user had the option to change the input response until satisfied. The program recognizes incorrectly formatted input and continues to prompt the user until a valid response is made.

BASIC TERMINOLOGY TEST RUN  
\*\*\*\*\*

THE FOLLOWING IS A LIST OF TERMS USED IN EXPERIMENTAL DESIGN.

ALIAS

EFFECT OF A FACTOR WHICH CANNOT BE DISTINGUISHED FROM  
THAT OF ANOTHER FACTOR.

ALPHA

IN A CENTRAL COMPOSITE DESIGN THE NON-ZERO CODED LEVEL  
VALUE OF A FACTOR AT AN AXIAL POINT.

AXIAL POINTS OR STAR POINTS

IN A CENTRAL COMPOSITE DESIGN FOR EACH FACTOR THERE  
ARE TWO CORRESPONDING AXIAL POINTS: THE GIVEN FACTOR  
HAS CODED LEVEL -ALPHA AT ONE POINT AND +ALPHA AT THE  
OTHER, WHEREAS ALL OTHER FACTORS HAVE CODED LEVEL ZERO  
AT THESE POINTS.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN  
\*\*\*\*\*

CENTER POINT

THE POINT IN A CENTRAL COMPOSITE DESIGN WHERE ALL N FACTORS  
HAVE CODED LEVEL ZERO.

CENTRAL COMPOSITE DESIGN

A COMBINATION OF A FULL OR FRACTIONAL TWO-LEVEL FACTORIAL  
DESIGN AND SOME ADDITIONAL EXPERIMENTAL POINTS SELECTED  
IN A PARTICULAR MANNER TO ALLOW THE DETERMINATION OF THE  
QUADRATIC ONE FACTOR EFFECTS. IT IS SPECIFICALLY INTENDED  
TO ALLOW DETERMINATION OF THE CONSTRAINTS USED IN DEFINING  
A QUADRATIC APPROXIMATION OF THE RESPONSE SURFACE.

CODED LEVEL

THE LEVEL OF A FACTOR TRANSLATED FROM THE TRUE QUANTITATIVE  
LEVEL USED FOR SIMPLIFYING CALCULATIONS.

CONFOUNDING  
 AN EXPERIMENTAL ARRANGEMENT IN WHICH CERTAIN EFFECTS CANNOT  
 BE DISTINGUISHED FROM OTHERS.

III RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN  
 \*\*\*\*\*

CORRELATION COEFFICIENT (PEARSON R)  
 THE SQUARE ROOT OF THE TOTAL VARIATION ACCOUNTED  
 FOR BY LINEAR REGRESSION.

CORRELATION INDEX R  
 THE SQUARE ROOT OF THE PROPORTION OF TOTAL VARIATION ACCOUNTED  
 FOR BY THE REGRESSION EQUATION OF THE DEGREE BEING FITTED TO THE DATA.

DEFINING CONTRAST  
 SELECTION OF EFFECTS TO BE CONFOUNDED.

DEGREES OF FREEDOM  
 IN THIS PROGRAM, ONE LESS THAN THE NUMBER OF VALUES TO COMPUTE  
 THE SUM OF SQUARES.

EFFECT  
 CHANGE IN RESPONSE DUE TO A CHANGE IN THE LEVEL OF A FACTOR.

III RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN  
 \*\*\*\*\*

EXPERIMENTAL MODEL  
 HYPOTHEZIZED EQUATION TO DESCRIBE THE RESPONSE AS A  
 FUNCTION OF THE TREATMENT.

EXPERIMENTAL TRIAL  
 ONE UNIT OF A COMPLETE EXPERIMENT, CONDUCTED WITH FACTORS AT  
 LEVELS DEFINED BY A SINGLE OBSERVATION VECTOR.



FACTORIAL EXPERIMENT  
AN EXPERIMENT IN WHICH ALL LEVELS OF EACH FACTOR IN THE  
EXPERIMENT ARE COMBINED WITH ALL LEVELS OF EVERY OTHER FACTOR.  
FRACTIONAL FACTORIAL  
AN EXPERIMENTAL DESIGN IN WHICH ONLY A FRACTION OF A  
COMPLETE FACTORIAL IS RUN.

HIT RETURN TO CONTINUE.

BASIC TERMINOLOGY TEST RUN  
\*\*\*\*\*

INTERACTION

AN INTERACTION BETWEEN TWO FACTORS MEANS THAT A CHANGE  
IN RESPONSE BETWEEN LEVELS OF ONE FACTOR IS NOT THE  
SAME FOR ALL LEVELS OF THE OTHER FACTOR.

MEAN SQUARE ERROR

SUM OF SQUARES OF THE ERROR DIVIDED BY THE NUMBER OF DEGREES  
OF FREEDOM FOR THE ERROR TERM.

MIXED LEVEL DESIGN

A FULL OR FRACTIONAL FACTORIAL DESIGN WHERE SOME FACTORS OF  
THE DESIGN HAVE A DIFFERENT NUMBER OF LEVELS THAN OTHER  
FACTORS OF THE DESIGN.

OBSERVATION VECTOR

PLANNED LEVEL OF EACH FACTOR FOR A SINGLE EXPERIMENTAL TRIAL.

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN  
\*\*\*\*\*

REAL WORLD LEVEL

THE TRUE QUANTITATIVE LEVEL OF A FACTOR THAT CORRESPONDS TO A  
CODED LEVEL.

REGRESSION

LINEAR -RESPONSE =  $A \cdot X_1 + B \cdot X_2 + C \cdot X_3 + \dots + Z \cdot X_N$   
QUADRATIC -RESPONSE =  $A \cdot X_1 + B \cdot X_2 + \dots + C \cdot X_1 \cdot X_2 + D \cdot X_1 \cdot X_3 + \dots + E \cdot X_1^2 + F \cdot X_2^2 + \dots + Z \cdot X_N^2$ .

REPLICATE

REPETITION OF OBSERVATION VECTORS APPLIED TO MULTIPLE  
EXPERIMENTAL TRIALS.

RESPONSE FUNCTION

THE FUNCTION  $F$  OR  $Y = F(X_1, X_2, \dots, X_N)$  WHERE THE LEVELS  
OF THE FACTORS ARE  $X_1, X_2, \dots, X_N$  AND THE RESPONSE IS  $Y$ .

RESPONSE SURFACE

THE SURFACE IN  $N+1$  DIMENSIONAL SPACE REPRESENTED BY THE  
EQUATION  $Y = F(X_1, X_2, \dots, X_N)$ .

HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

ROOT SUM SQUARE (RSS)

THE SQUARE ROOT OF THE SUM OF THE SQUARES

R-SQUARED

SMALL R-SQUARED -SEE CORRELATION COEFFICIENT.  
BIG R-SQUARED - SEE CORRELATION INDEX.

TRIAL

A SINGLE SET OF FACTOR VALUES APPLIED TO THE EXPERIMENTAL  
SUBJECT FOR WHICH THE RESPONSE IS MEASURED.

DO YOU WANT TO REVIEW THESE DEFINITIONS?

NO

BASIC TERMINOLOGY TEST RUN

\*\*\*\*\*

DO YOU WANT TO RERUN THIS SEGMENT?

NO

### PROBLEM DEFINITION

Segment 2--Problem Definition--is used by the experimenter to define the particular problem being studied. A new menu is displayed allowing the user to select one of four options.

### BASIC FACTORIAL DESIGNS

Option 1--Basic Factorial Designs--allows the user to define full or fractional factorial designs for 2, 3, or 5 levels. This definition includes: (1) the number of factors, (2) the number of levels per each factor, (3) the number of experimental trials available, and (4) aliasing information.

Since the process of specifying the defining contrasts that determine the aliasing is critical, the user is offered assistance in the definition of these aliases. In this help section, the program can provide certain predefined designs (see example 1). These designs are described in Volume I of this report.

If the predefined alias set is unacceptable or if one is not available, the user is given help (for 2 level designs only) in developing a "good" alias set by taking the predefined design and either by deleting factors (see example 2) or by adding factors (see example 3) arrives at a new option design. If the user is still not satisfied with the design, the defining contrasts can be specified as input values.

Care should be taken that the members of the alias set are linearly independent, since if any member of the alias set is a linear combination of the other members, the experimental block will contain more than the desired number of observations. The program will check for independence of the alias set, and give the user an opportunity to redefine the set.

The user may also have the total alias set displayed. (The total alias set is constructed by forming all possible combinations of the original defining contrasts specified by the user.) The alias set may be redefined until the design is acceptable to the user.

The user may also have the total alias set displayed. (The total alias set is constructed by forming all possible combinations of the original defining contrasts specified by the user.) The alias set may be redefined until the design is acceptable to the user.

PROBLEM DEFINITION

BASIC FACTORIAL DESIGNS

EXAMPLE 1

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

- 1. BASIC TERMINOLOGY
- 2. PROBLEM DEFINITION
- 3. ACTUAL EXPERIMENTAL DESIGN
- 4. EXPERIMENTAL REFINEMENT
- 5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.  
PROBLEM DEFINITION AND DESIGN TEST

PROBLEM DEFINITION AND DESIGN TEST  
\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

- 1. BASIC FACTORIAL DESIGNS
- 2. MIXED LEVEL DESIGNS
- 3. CENTRAL COMPOSITE DESIGNS
- 4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS  
NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.  
HOW MANY LEVELS FOR THE FACTORS? (2,3, or 5)  
2  
HOW MANY FACTORS ARE INVOLVED?  
YOU MAY CHOOSE UP TO 20.

5



HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?

16

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?

Y

5 FACTORS ARE PRESENT. IS THIS CORRECT?

Y

16 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?

Y

31 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.

OF THESE:

5 ARE MAIN EFFECTS.

10 ARE FIRST ORDER INTERACTIONS.

16 ARE HIGHER ORDER INTERACTIONS.

HIT RETURN WHEN READY TO CONTINUE

PROBLEM DEFINITION AND DESIGN TEST

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?

THIS MUST BE SELECTED FROM THE VALUES

2 4 8 16 32

16

IS THIS WHAT YOU WANT: 16

YES

THIS IS A 1/2 FRACTIONAL FACTORIAL DESIGN.

FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED 1 LINEARLY INDEPENDENT  
DEFINING CONTRAST(S).

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.

THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO  
DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES THE FACTOR  
IS AT ITS LOW LEVEL.

THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

AC MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL,  
AND FACTOR C AT ITS HIGH LEVEL.

WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
YES

PROBLEM DEFINITION AND DESIGN TEST  
\*\*\*\*\*

BASED ON THE EXPERIMENTAL DESCRIPTION YOU HAVE GIVEN ABOUT THE NUMBER OF  
FACTORS, THE FACTOR LEVELS, AND THE NUMBER OF TRIALS TO BE USED,  
THE FOLLOWING DESIGN IS FEASIBLE: DESIGN 2. 5, 16, 1/2 REPLICATE  
ALIAS DEFINITION:

I=ABCDE

DO YOU WANT TO USE THIS DESIGN?  
YES

THE DEFINING CONTRAST SET DEFINED CONTAINS 1 INDEPENDENT MEMBER (S)

DO YOU WANT TO SEE THE TOTAL ALIAS SET?  
YES

TOTAL ALIAS SET  
ABCDE

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?  
NO

PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE.

AD-A124 303

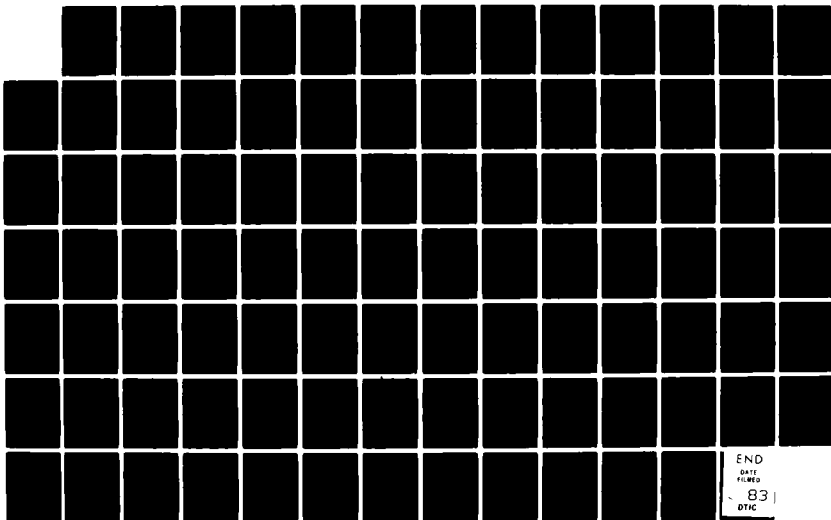
THE USER-ASSISTED AUTOMATED EXPERIMENTAL (TEST) DESIGN  
PROGRAM (AED): VERSION II(U) SYSTEM DEVELOPMENT CORP  
DAYTON OHIO E G MEYER ET AL. JAN 83 AFAMRL-TR-82-100  
F33615-79-C-0505

22

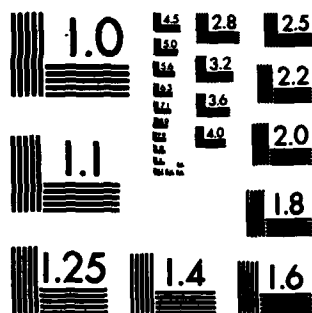
UNCLASSIFIED

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END  
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83  
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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

EXAMPLE 2

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT \_NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.  
HELP DEMONSTRATION

HELP DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS

NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.  
HOW MANY LEVELS FOR THE FACTORS? (2,3, or 5)  
2

HOW MANY FACTORS ARE INVOLVED?  
YOU MAY CHOOSE UP TO 20.  
3

HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?

2

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?

Y

3 FACTORS ARE PRESENT. IS THIS CORRECT?

Y

2 EXPERIMENTAL TRIALS ARE AVAILABLE, IS THIS CORRECT?

Y

7 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.  
OF THESE:

3 ARE MAIN EFFECTS.

3 ARE FIRST ORDER INTERACTIONS.

1 ARE HIGHER ORDER INTERACTIONS.

HIT RETURN WHEN READY TO CONTINUE.

HELP DEMONSTRATION

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?  
THIS MUST BE SELECTED FROM THE VALUES?

2 4 8

2

IS THIS WHAT YOU WANT: 2

Y

THIS IS A 1/4 FRACTIONAL FACTORIAL DESIGN.  
FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED 2  
LINEARLY INDEPENDENT DEFINING CONTRAST (s).

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.  
THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO  
DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES THE FACTOR IS AT ITS LOW LEVEL. THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

AC MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL, AND FACTOR C AT ITS HIGH LEVEL.

WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?

Y

HELP DEMONSTRATION

\*\*\*\*\*

BASED ON THE EXPERIMENTAL DESCRIPTION YOU HAVE GIVEN ABOUT THE NUMBER OF FACTORS, THE FACTOR LEVELS, AND THE NUMBER OF TRIALS TO BE USED, NO PREDEFINED DESIGN IS AVAILABLE. SEE "INTRODUCTION TO EXPERIMENTAL DESIGN AND THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM", VOL. I, SECTION 4 FOR MORE DETAIL.

DO YOU WANT HELP IN GENERATING A REALIZABLE

2 LEVEL ALIAS SET?

YES

HELP DEMONSTRATION

\*\*\*\*\*

GENERALLY, IN A TWO LEVEL EXPERIMENT INVOLVING ANY NUMBER OF FACTORS (REPRESENTED BY THE LETTER N) IT IS DESIRED TO ISOLATE ALL SINGLE FACTORS AND TWO FACTOR INTERACTIONS. THIS IS UNDER THE ASSUMPTION THAT THREE FACTOR AND HIGHER ORDER INTERACTIONS ARE INSIGNIFICANT.

TO OBTAIN THE LARGEST DEGREE OF ISOLATION FOR A GIVEN EXPERIMENT, ALL MEMBERS OF THE ALIAS SET SHOULD CONTAIN APPROXIMATELY THE SAME NUMBER OF HIGH LEVEL FACTORS.

HIT RETURN TO CONTINUE



天  
地  
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天  
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人  
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**FOR A 1/4 REPLICATE OF A 2 LEVEL DESIGN WITH 5 FACTORS, THE TOTAL ALIAS SET OF THIS DESIGN MIGHT BE:**

ABE CDE ABCD

**IN NOTATION FORM THIS BECOMES:**

$$2(3) + 1(4)$$

## HELP DEMONSTRATION

[illegible]

		COLUMN NUMBER							
		1	2	3	4	5	6	7	
1	DEFINING CONTRAST NO.	1	1	1	1	1	0	0	5 FACTORS
2	DEFINING CONTRAST NO.	2	1	1	0	0	1	1	5 FACTORS
1x2	PRODUCT OF 1 AND 2	-	DEFG	0	0	1	1	1	4 FACTORS

**THIS IS A 1(4) + 2(5) DESIGN.**

**QUIT RETURN TO CCNTINUE.**

HELP DEMONSTRATION  
\*\*\*\*\*

A NEW DESIGN FOR SIX (6) FACTORS CAN BE GENERATED FROM THIS DESIGN BY DELETING ONE OF THE COLUMNS. THE BEST THEORETICAL DESIGN FOR SIX (6) FACTORS IS A 3(4) DESIGN. KNOWING THIS WILL ALLOW US TO MAKE A BETTER JUDGEMENT ABOUT WHICH COLUMN TO DELETE. REMOVING ANYONE OF THE COLUMNS SHOWN WILL RESULT IN THE CORRESPONDING DESIGN.

COLUMN REMOVED	RESULTING DESIGN
1	3(4)
2	3(4)
3	3(4)
4	1(3) + 1(4) + 1(5)
5	1(3) + 1(4) + 1(5)
6	1(3) + 1(4) + 1(5)
7	1(3) + 1(4) + 1(5)

18

HIT RETURN TO CONTINUE

HELP DEMONSTRATION  
\*\*\*\*\*

BY REMOVING COLUMN 1, 2, or 3 THE BEST THEORETICAL DESIGN WILL BE OBTAINED.

	COLUMN NUMBER
1 DEFINING CONTRAST NO. 1 - ABCD	1 2 3 4 5 6
2 DEFINING CONTRAST NO. 2 - ABCE	1 1 1 1 0 0
1x2 PRODUCT OF 1 AND 2 - CDEF	1 1 0 0 1 1
	0 0 1 1 1 1
	4 FACTORS
	4 FACTORS
	4 FACTORS

THIS IS A 3(4) DESIGN

HIT RETURN TO CONTINUE

HELP DEMONSTRATION  
\*\*\*\*\*

AS ANOTHER EXAMPLE, IF TWO (2) DEFINING CONTRASTS, ABE AND CDE, ARE USED IN A FIVE (5) FACTOR PROBLEM, THE FOLLOWING ALIAS SET IS GENERATED:

	COLUMN NUMBER					
	1	2	3	4	5	
1 DEFINING CONTRAST NO. 1 - ABE	1	1	0	0	1	3FACTORS
2 DEFINING CONTRAST NO. 2 - CDE	0	0	1	1	1	3FACTORS
1x2 PRODUCT OF 1 and 2 - ABCD	1	1	1	1	0	4FACTORS

THIS IS A 2(3) + 1(4) DESIGN

HIT RETURN TO CONTINUE

HELP DEMONSTRATION  
\*\*\*\*\*

A NEW DESIGN FOR SIX (6) FACTORS CAN BE GENERATED FROM THIS DESIGN BY ADDING ANOTHER COLUMN. THE BEST THEORETICAL DESIGN FOR SIX (6) FACTORS IS A 3(4) DESIGN. KNOWING THIS WILL ALLOW US TO MAKE A BETTER JUDGEMENT ABOUT WHICH ROWS SHOULD BE INCREASED.

THIS MIGHT BE DONE AS FOLLOWS:

DEFINING CONTRAST PRODUCTS	ORIGINAL DESIGN	ADDED COLUMN	NUMBER OF FACTORS PER ROW
1	1 2 3 4 5	1	4 FACTORS
2	1 1 0 0 1	1	4 FACTORS
1x2	0 0 1 1 1	0	4 FACTORS
	1 1 1 1 0		

HIT RETURN TO CONTINUE.

HELP DEMONSTRATION  
\*\*\*\*\*

BY INCREASING DEFINING CONTRAST 1 AND 2, THE BEST THEORETICAL  
DESIGN WILL BE OBTAINED.

		1	2	3	4	5	6	
1	DEFINING CONTRAST NO. 1	-	A	B	E	F		4 FACTORS
2	DEFINING CONTRAST NO. 2	-	C	D	E	F		4 FACTORS
1x2	PRODUCT OF 1 and 2	-	A	B	C	D		4 FACTORS

THIS IS A 3(4) DESIGN.

HIT RETURN TO CONTINUE

HELP DEMONSTRATION  
\*\*\*\*\*

YOUR PRESENT DESIGN CONSISTS OF 2 LEVELS,  
3 FACTORS, AND 2 DEFINING CONTRASTS.

A REALIZABLE DESIGN CAN BE CONSTRUCTED BY STARTING WITH A STORED DESIGN.  
STORED DESIGNS FOR THE PRESENT NUMBER OF DEFINING CONTRASTS ARE AS FOLLOWS:

NO. OF DEFINING CONTRASTS	=	2	STORED DESIGNS FOR N (NO. OF FACTORS)	=	5
					= 6
					= 7
					= 8
					= 9
					= 10

YOU MAY ENTER A VALUE OF N FROM THE STORED DESIGNS SMALLER THAN YOUR DESIRED  
VALUE OF N AND BUILD UP TO THE DESIRED VALUE OF N OR ENTER A LARGER VALUE OF  
N FROM THE STORED DESIGNS AND REDUCE TO THE DESIRED VALUE OF N.  
ENTER THE STORED DESIGN VALUE OF N YOU WISH TO START WITH.

IS THIS THE NUMBER YOU WANT? 5  
Y

HELP DEMONSTRATION  
\*\*\*\*\*

DEFINING CONTRAST PRODUCTS	TOTAL ALIAS SET					NUMBER OF FACTORS PER ROW
	COLUMN NUMBER					
1	1	2	3	4	5	3
	0	0	1	1	1	
2	1	1	0	0	1	3
1 2	1	1	1	1	0	4

THE ALIAS SET OF THE PRESENT DESIGN CONSISTS OF:

$2(3) + 1(4)$

DO YOU WISH TO REDUCE N?  
Y

HELP DEMONSTRATION  
\*\*\*\*\*

THE BEST THEORETICAL ALIAS SET WOULD CONTAIN:

$1(2) + 2(3)$

AN ALTERNATIVE BEST THEORETICAL ALIAS SET WOULD CONTAIN:

$2(2) + 1(4)$

THE BEST THEORETICAL DESIGN MAY OR MAY NOT BE ACHIEVABLE.

REMOVING ANY GIVEN COLUMN WILL RESULT IN THE FOLLOWING DESIGNS:

COLUMN REMOVED	RESULTING DESIGN
1	$1(2) + 2(3)$
2	$1(2) + 2(3)$
3	$1(2) + 2(3)$
4	$1(2) + 2(3)$

HIT RETURN TO CONTINUE

5 2(2) + 1(4)

WHICH COLUMN DO YOU WISH REMOVED?

1

IS THIS THE NUMBER YOU WANT: 1

Y

HELP DEMONSTRATION

\*\*\*\*\*

DEFINING CONTRAST PRODUCT	TOTAL ALIAS SET				NUMBER OF FACTORS PER ROW
	COLUMN NUMBER				
1	1	2	3	4	3
	0	1	1	1	
2	1	0	0	1	2
1 2	1	1	1	0	3

THE ALIAS SET OF THE PRESENT DESIGN CONSISTS OF:

1(2) + 2(3)

DO YOU WISH TO REDUCE N?

Y

HELP DEMONSTRATION

\*\*\*\*\*

THE BEST THEORETICAL ALIAS SET WOULD CONTAIN:

3(2) FACTOR TERM

THE BEST THEORETICAL DESIGN MAY OR MAY NOT BE ACHIEVABLE.

REMOVING ANY GIVEN COLUMN WILL RESULT IN THE FOLLOWING DESIGNS:

COLUMN REMOVED	RESULTING DESIGN
1	1(1) + 1(2) + 1(3)
2	3(2)
3	3(2)
4	1(1) + 1(2) + 1(3)

WHICH COLUMN DO YOU WISH REMOVED?

2  
IS THIS THE NUMBER YOU WANT? 2  
Y

HELP DEMONSTRATION

\*\*\*\*\*

DEFINING CONTRAST PRODUCT	TOTAL ALIAS SET COLUMN NUMBER	NUMBER OF FACTORS PER ROW
1	1 2 3	2
2	0 1 1	2
1 2	1 0 1	2
	1 1 0	

THE ALIAS SET OF THE PRESENT DESIGN CONSISTS OF:

3(2)

DO YOU WISH TO REDUCE N?

N

DO YOU WISH TO INCREASE N?

N

THE DEFINING CONTRAST SET DEFINED CONTAINS 2 INDEPENDENT MEMBER(S)

DO YOU WANT TO SEE THE TOTAL ALIAS SET?

YES

TOTAL ALIAS SET

BC  
AC  
AB

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?

N

PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE.



EXAMPLE 3

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.

HELP DEMONSTRATION

HELP DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS

NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS: (2, 3, or 5)

2

HOW MANY FACTORS ARE INVOLVED?

YOU MAY CHOOSE UP TO 20.

6

HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?

16

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?

Y

6 FACTORS ARE PRESENT. IS THIS CORRECT?

Y

16 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?

Y

63 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.  
OF THESE:

6 ARE MAIN EFFECTS

15 ARE FIRST ORDER INTERACTIONS

41 ARE HIGHER ORDER INTERACTIONS

HIT RETURN WHEN READY TO CONTINUE

HELP DEMONSTRATION

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?

THIS MUST BE SELECTED FROM THE VALUES

2 4 8 16 32 64

16

IS THIS WHAT YOU WANT: 16

Y

THIS IS A 1/4 FRACTIONAL FACTORIAL DESIGN. FOR AN EXPERIMENT OF  
THIS SIZE, YOU WILL NEED 2 LINEARLY INDEPENDENT DEFINING CONTRAST(S).

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.  
THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING  
NOTATION IS USED TO DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR  
ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER  
INDICATES THE FACTOR IS AT ITS LOW LEVEL.  
THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

A C MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL,  
AND FACTOR C AT ITS HIGH LEVEL.

WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
Y

HELP DEMONSTRATION

\*\*\*\*\*

BASED ON THE EXPERIMENTAL DESCRIPTION YOU HAVE GIVEN ABOUT  
THE NUMBER OF FACTORS, THE FACTOR LEVELS, AND THE NUMBER OF TRIALS  
TO BE USED, THE FOLLOWING DESIGN IS FEASIBLE: DESIGN 2.6.16 1/4 REPLICATE  
ALIAS DEFINITION:

I - ABCE - ABDF

DO YOU WANT TO USE THIS DESIGN:

NO

DO YOU WANT HELP IN GENERATING A REALIZABLE 2 LEVEL ALIAS SET?

YES

HELP DEMONSTRATION

\*\*\*\*\*

GENERALLY, IN A TWO LEVEL EXPERIMENT INVOLVING ANY NUMBER OF FACTORS (REPRESENTED BY THE LETTER N) IT IS DESIRED TO ISOLATE ALL SINGLE FACTORS AND TWO FACTOR INTERACTIONS. THIS IS UNDER THE ASSUMPTION THAT THREE FACTOR AND HIGHER ORDER INTERACTIONS ARE INSIGNIFICANT.

TO OBTAIN THE LARGEST DEGREE OF ISOLATION FOR A GIVEN EXPERIMENT, ALL MEMBERS OF THE ALIAS SET SHOULD CONTAIN APPROXIMATELY THE SAME NUMBER OF HIGH LEVEL FACTORS.

HIT RETURN TO CONTINUE

HELP DEMONSTRATION

\*\*\*\*\*

A USEFUL NOTATION HAS BEEN DEVELOPED TO DESCRIBE THE COMPOSITION OF THE TOTAL ALIAS SET. AN EXAMPLE OF THIS NOTATION FOLLOWS:

FOR A 1/4 REPLICATE OF A 2 LEVEL DESIGN WITH 5 FACTORS, THE TOTAL ALIAS SET OF THIS DESIGN MIGHT BE:

ABE  
CDE  
ABCD

THIS ALIAS SET CONTAINS THREE (3) MEMBERS, TWO (2) MEMBERS WITH THREE (3) HIGH LEVEL FACTORS EACH AND ONE (1) MEMBER WITH FOUR (4) HIGH LEVEL FACTORS.

IN NOTATION FORM THIS BECOMES:

2(3) + 1(4)

WOULD YOU LIKE TO SEE AN EXAMPLE OF GENERATING A NEW DESIGN?  
NO

# HELP DEMONSTRATION

\*\*\*\*\*

YOUR PRESENT DESIGN CONSISTS OF 2 LEVELS,  
6 FACTORS, AND 2 DEFINING CONTRASTS.

A REALIZABLE DESIGN CAN BE CONSTRUCTED BY STARTING WITH A  
STORED DESIGN. STORED DESIGNS FOR THE PRESENT NUMBER OF DEFINING CONTRASTS  
ARE AS FOLLOWS:

NO. OF DEFINING CONTRASTS - 2 STORED DESIGNS FOR N (NO. OF FACTORS ) - 5  
- 6  
- 7  
- 8  
- 9  
-10

YOU MAY ENTER A VALUE OF N FROM THE STORED DESIGNS SMALLER THAN YOUR DESIRED  
VALUE OF N AND BUILD UP TO THE DESIRED VALUE OF N OR ENTER A LARGER VALUE OF  
N FROM THE STORED DESIGNS AND REDUCE TO THE DESIRED VALUE OR N.

ENTER THE STORED DESIGN VALUE OF N YOU WISH TO START WITH.

5  
IS THIS THE NUMBER YOU WANT? 5  
Y

# HELP DEMONSTRATION

\*\*\*\*\*

DEFINING CONTRAST	TOTAL ALIAS SET					NUMBER OF FACTORS
PRODUCTS	COLUMN NUMBER					PER ROW
1	1	2	3	4	5	3
2	0	0	1	1	1	3
1 2	1	1	0	0	1	4
	1	1	1	1	0	

THE ALIAS SET OF THE PRESENT DESIGN CONSISTS OF:

2(3) + 1(4)

DO YOU WISH TO REDUCE N?  
NO

DO YOU WISH TO INCREASE N?  
YES

HELP DEMONSTRATION  
\*\*\*\*\*

THE BEST THEORETICAL ALIAS SET WOULD CONTAIN:

3(4)

THE BEST THEORETICAL DESIGN MAY OR MAY NOT BE ACHIEVABLE.

SELECT A DEFINING CONTRAST PRODUCT YOU WISH TO CHANGE. IF YOU  
WISH TO INCREASE IT, ENTER A 1.

IF YOU DO NOT WISH TO INCREASE IT, ENTER A 0.

DEFINING CONTRAST PRODUCTS MUST BE OF THE FORM:

DEFINING CONTRAST PRODUCT = 0 or 1.

ENTER THE DEFINING CONTRAST PRODUCT. BE SURE TO LEAVE A SPACE BETWEEN  
NUMBERS AS IN THE FOLLOWING EXAMPLE: 1 2 3

1

NOW ENTER A 0 OR 1 FOR THE ABOVE DEFINING CONTRAST PRODUCT.

1

1 - 1

IS THIS CORRECT?  
YES

HELP DEMONSTRATION  
\*\*\*\*\*

DEFINING CONTRAST PRODUCT	ORIGINAL DESIGN	ADDED COLUMN	NUMBER OF FACTORS PER ROW
1	1 2 3 4 5		
2	0 0 1 1 1	1	4
1 2	1 1 0 0 1		3
	1 1 1 1 0		4

SELECT A DEFINING CONTRAST PRODUCT YOU WISH TO CHANGE.

IF YOU WISH TO INCREASE IT, ENTER A 1.

IF YOU DO NOT WISH TO INCREASE IT, ENTER A 0.

DEFINING CONTRAST PRODUCTS MUST BE OF THE FORM:

DEFINING CONTRAST PRODUCT = 0 or 1.

ENTER THE DEFINING CONTRAST PRODUCT. BE SURE TO LEAVE

A SPACE BETWEEN NUMBERS AS IN THE FOLLOWING EXAMPLE: 1 2 3

1 2

NOW ENTER A 0 OR 1 FOR THE ABOVE DEFINING CONTRASTS PRODUCT.

0

1 2 - 0

IS THIS CORRECT?

YES

HELP DEMONSTRATION

\*\*\*\*\*

DEFINING CONTRAST PRODUCT	ORIGINAL DESIGN	ADDED COLUMN	NUMBER OF FACTORS PER ROW
1	1 2 3 4 5		
2	0 0 1 1 1	1	4
1 2	1 1 0 0 1	1	4
	1 1 1 1 0	0	4

HIT RETURN TO CONTINUE



# HELP DEMONSTRATION

\*\*\*\*\*

DEFINING CONTRAST PRODUCT	TOTAL ALIAS SET						NUMBER OF FACTORS PER ROW
	COLUMN NUMBER						
1	1	2	3	4	5	6	
	0	0	1	1	1	1	4
2	1	1	0	0	1	1	4
1 2	1	1	1	1	0	0	4

THE ALIAS SET OF THE PRESENT DESIGN CONSISTS OF:

3(4)

DO YOU WISH TO REDUCE N?

1 NO

DO YOU WISH TO INCREASE N?

3 NO

THE DEFINING CONTRAST SET DEFINED CONTAINS 2 INDEPENDENT MEMBER (s).  
DO YOU WANT TO SEE THE TOTAL ALIAS SET?

NO

HIT RETURN WHEN READY TO CONTINUE

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?

NO

PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE.

### MIXED LEVEL DESIGNS

Option 2--Mixed Level Designs allow the user to define full or fractional factorial designs where some factors of the design have a different number of levels than other factors of the design. A menu is displayed, but at this time only the 2 level crossed with the 3 level case has been developed (see the following example). The mixed level design is treated like two separate designs, one for the factors at 2 levels and one for the factors at 3 levels. Then the resulting observation blocks from the two separate designs are combined to produce a mixed level observation block.

The displayed menu also allows the user to exit if the 2 x 3 mixed level is not desired.

MIXED LEVEL DESIGNS

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN

2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.

MIXED LEVEL DESIGN DEMONSTRATION

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MIXED LEVEL DESIGN DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN

2

YOUR ENTRY WAS: 2--MIXED LEVEL DESIGNS

THE MIXED LEVEL DESIGN SEGMENT ALLOWS THE EXPERIMENTER  
TO COMBINE EXPERIMENTAL DESIGNS OF DIFFERENT LEVELS,  
E.G. 2 LEVEL DESIGNS WITH 3 LEVEL DESIGNS, ETC.

THE EASIEST APPROACH TO COMBINING DESIGNS OF DIFFERENT LEVELS IS TO CONSIDER EACH LEVEL INDEPENDENTLY BY PERFORMING AN OPTIMUM FRACTIONATION FOR THAT LEVEL FOLLOWED BY A CROSSING MULTIPLICATION OF THE OBSERVATION BLOCKS FOR EACH LEVEL.

THE AVAILABLE DESIGNS ARE:

1. 2X3
2. EXIT

ENTER THE DESIGN NUMBER YOU WANT AND HIT RETURN.

1

YOUR ENTRY WAS: 1--2X3

HOW MANY FACTORS ARE INVOLVED IN THE 2 LEVEL PORTION OF THE MIXED LEVEL DESIGN?

YOU MAY CHOOSE UP TO 18.

1

FOR 2 LEVELS, THERE ARE 1 FACTORS.

IS THIS WHAT YOU WANT?

YES

HOW MANY FACTORS ARE INVOLVED IN THE 3 LEVEL PORTION OF THE MIXED LEVEL DESIGN?

YOU MAY CHOOSE UP TO 11.

2

FOR 3 LEVELS, THERE ARE 2 FACTORS.

IS THIS WHAT YOU WANT?

YES

HOW MANY TOTAL EXPERIMENTAL TRIALS ARE TO BE RUN FOR THE MIXED LEVEL DESIGN? THIS MUST BE SELECTED FROM THE VALUES:

2	3	6	9	18
---	---	---	---	----

6

IS THIS WHAT YOU WANT: 6  
YES

THIS IS A 1/3 FRACTIONAL FACTORIAL DESIGN.

HIT RETURN TO CONTINUE

MIXED LEVEL DESIGN DEMONSTRATION  
\*\*\*\*\*

FOR A MIXED LEVEL DESIGN YOU WILL HAVE TO DEFINE THE  
ALIAS SET FOR ONE LEVEL AT A TIME.

FOR THE 2 LEVEL PORTION OF THE MIXED LEVEL DESIGN, YOU  
WILL NEED 1 LINEARLY INDEPENDENT DEFINING CONTRAST (S).

HIT RETURN TO CONTINUE

MIXED LEVEL DESIGN DEMONSTRATION  
\*\*\*\*\*

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.  
THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING  
NOTATION IS USED TO DESCRIBE THE FACTORS IN A PARTICULAR TRIAL  
OR ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES  
THE FACTOR IS AT ITS LOW LEVEL.

THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

AC MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL,  
AND FACTOR C AT ITS HIGH LEVEL.  
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
NO

YOU MAY DEFINE CONTRASTS BY DESCRIBING WHICH EFFECTS  
ARE TO BE CONFOUNDED. FOR EXAMPLE, AB=CD. ANOTHER  
(MORE COMMON) WAY IS TO DEFINE THE ALIASING IN TERMS  
OF THE IDENTITY EFFECT. FOR EXAMPLE, I = ABCD

REMEMBER THAT THE DEFINING CONTRAST MUST BE OF THE FORM EFFECT 1 = EFFECT 2  
THE FIRST CONTRAST EFFECT IS INPUT THEN THE SECOND.

DEFINING CONTRAST NO. 1 FIRST TERM

I

DEFINING CONTRAST NO. 1 SECOND TERM

A

IS THIS CORRECT?

I

YES

THE DEFINING CONTRAST SET DEFINED CONTAINS 1 INDEPENDENT MEMBER(s).

DO YOU WANT TO SEE THE TOTAL ALIAS SET?

YES

TOTAL ALIAS SET

A

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?

NO

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS AND FIRST ORDER  
INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER  
INTERACTIONS. MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE  
MAIN EFFECTS THAT ARE CONFOUNDED WITH OTHER MAIN EFFECTS ARE FLAGGED  
WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE.

# EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	0 M	A

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE.

## MIXED LEVEL DESIGN DEMONSTRATION

\*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

HIT RETURN TO CONTINUE.

## MIXED LEVEL DESIGN DEMONSTRATION

\*\*\*\*\*

FOR A MIXED LEVEL DESIGN YOU WILL HAVE TO DEFINE THE ALIAS  
 SET FOR ONE LEVEL AT A TIME.

FOR THE 3 LEVEL PORTION OF THE MIXED LEVEL DESIGN, YOU WILL  
 NEED 1 LINEARLY INDEPENDENT DEFINING CONTRAST(s).

HIT RETURN TO CONTINUE.

## MIXED LEVEL DESIGN DEMONSTRATION

\*\*\*\*\*

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.  
 THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT. SINCE EACH FACTOR  
 MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO DESCRIBE  
 THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.

FOR A 3 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER IS THE LOW LEVEL,  
 THE PRESENCE OF THE LETTER INDICATES THE INTERMEDIATE LEVEL, WHILE THE LETTER  
 FOLLOWED BY A 2 INDICATES THE HIGH LEVEL.

A2C MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL, AND FACTOR C



AT ITS INTERMEDIATE LEVEL.  
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
NO.

YOU MAY DEFINE CONTRASTS BY DESCRIBING WHICH EFFECTS ARE TO BE CONFOUNDED.  
FOR EXAMPLE, AB = CD.  
ANOTHER (MORE COMMON) WAY IS TO DEFINE THE ALIASING IN TERMS OF THE  
IDENTITY EFFECT. FOR EXAMPLE, I = ABCD.

REMEMBER THAT THE DEFINING CONTRAST MUST BE OF THE FORM EFFECT 1 = EFFECT 2  
THE FIRST CONTRAST EFFECT IS INPUT THEN THE SECOND.

DEFINING CONTRAST NO. 1 FIRST TERM

I

DEFINING CONTRAST NO. 1 SECOND TERM

A2B2

IS THIS CORRECT?

I

-A2B2

YES

-121

THE DEFINING CONTRAST SET DEFINED CONTAINS 1 INDEPENDENT MEMBER(s).

DO YOU WANT TO SEE THE TOTAL ALIAS SET?

YES

TOTAL ALIAS SET

A2B2

AB

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?

NO

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS AND FIRST ORDER INTERACTIONS  
CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER INTERACTION. MAIN EFFECTS  
ARE MARKED WITH THE LETTER M, AND THOSE MAIN EFFECTS THAT ARE CONFOUNDED WITH  
OTHER MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE.

# EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
1	1	0 M *	A
0	1	0	AB
2	0	0	AB2
1	1	0 M *	A2
2	0	0	A2B
0	1	0	A2B2
1	1	0 M *	B
1	1	0 M *	B2

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE

MIXED LEVEL DESIGN DEMONSTRATION  
\*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
IN THE DATA COLLECTION PROCESS.  
FOR THE 2 LEVEL PER FACTOR PORTION ON THE EXPERIMENT,  
0 AND 1 REPRESENT THE LOW AND HIGH FACTOR VALUES.  
FOR THE 3 LEVEL PER FACTOR PORTION OF THE EXPERIMENT,  
0, 1, AND 2 REPRESENT THE LOW, INTERMEDIATE, AND  
HIGH FACTOR VALUES.

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR  
TO BE PRINTED ON THE LINE PRINTER?

NO

HIT RETURN TO CONTINUE.

# MIXED LEVEL DESIGN DEMONSTRATION

\*\*\*\*\*

NOW THAT THE DESIGNS FOR EACH LEVEL HAVE BEEN DEFINED, THE BASIC EXPERIMENTAL OBSERVATION BLOCK WILL BE FORMED BY COMBINING THE SEPARATE DESIGNS. SEE "INTRODUCTION TO EXPERIMENTAL DESIGN AND THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM", VOL. 1, SECTION 1/4- FOR MORE DETAIL. THE LETTER DESIGNATIONS FOR THE FACTORS HAVE BEEN CHANGED TO UNIQUELY SPECIFY THEM. FOR THE FOLLOWING FACTORS THE FIRST 1 WILL BE AT THE 2 LEVEL AND THE LAST 2 WILL BE AT THE 3 LEVEL.

## BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABC  
000  
012  
021  
102  
111  
120

HIT RETURN TO CONTINUE

### CENTRAL COMPOSITE DESIGNS

Option 3--Central Composite Designs allow the user to add more observation vectors to the basic factorial experiments in order that a quadratic approximation to the response surface of the experimental region can be made. The user can choose a rotatable or non-rotatable central composite design. There are also two methods for specifying the real world levels. One is to specify the real world range for each factor. The other is to specify the real world levels of each factor in the basic factorial design.

Central composite designs can be defined in one of two ways. The user may use program segment 2 - option 1 and program segment 3 to define the basic factorial portion of the central composite design, then enter segment 2 - option 3 to complete the additional experiments (see example 1).

Or the user may enter directly into segment 2 - option 3 in which case, the program will prompt the user for the basic factorial definition (see example 2).

### EXIT

Option 4--Exit allows the user to exit from the problem definition segment.

CENTRAL COMPOSITE DESIGNS

EXAMPLE 1

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.  
CENTRAL COMPOSITE DEMONSTRATION

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS

NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS? (2,3, OR 5)  
2

HOW MANY FACTORS ARE INVOLVED?  
YOU MAY CHOOSE UP TO 20.

3 HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?  
 8  
 Y 2 LEVELS FOR THE FACTORS. IS THIS CORRECT?  
 Y 3 FACTORS ARE PRESENT. IS THIS CORRECT?  
 Y 8 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?  
 7 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.  
 OF THESE:  
 3 ARE MAIN EFFECTS.  
 3 ARE FIRST ORDER INTERACTIONS.  
 1 ARE HIGHER ORDER INTERACTIONS.

HIT RETURN WHEN READY TO CONTINUE

# CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?  
 THIS MUST BE SELECTED FROM THE VALUES:

2 4 8

8 IS THIS WHAT YOU WANT: 8  
 Y

THIS IS A FULL FACTORIAL DESIGN.  
 FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED  
 0 LINEARLY INDEPENDENT DEFINING CONTRAST(S).  
 PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE.



CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN

4

YOUR ENTRY WAS: 4--EXIT

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN

3

YOUR ENTRY WAS: 3--ACTUAL EXPERIMENTAL DESIGN  
WOULD YOU LIKE TO CHANGE THE RUN I.D.?

NO

NO ALIASING IS USED IN THIS DESIGN.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR

THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN IN THE DATA COLLECTION PROCESS.

FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT THE LOW AND HIGH FACTOR VALUES.

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR TO BE PRINTED ON THE LINE PRINTER?

NO

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABC
000
001
010
011

100  
101  
110  
111

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN

2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

WOULD YOU LIKE TO CHANGE THE RUN I.D.?

NO

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN

3

YOUR ENTRY WAS: 3--CENTRAL COMPOSITE DESIGNS

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

AN N-FACTOR CENTRAL COMPOSITE DESIGN CONSISTS OF THREE COMPONENTS.

1. A FULL OR FRACTIONAL REPLICATE OF A TWO-LEVEL FACTORIAL DESIGN, WHERE FOR EACH FACTOR THE TWO CODED LEVELS ARE -1 and 1.
2. TWO CORRESPONDING AXIAL POINTS FOR EACH FACTOR WITH CODED LEVEL -ALPHA AT ONE POINT, +ALPHA AT THE OTHER AND WITH ALL OTHER FACTORS HAVING CODED LEVEL ZERO AT THESE POINTS.
3. THE CENTER POINT OF EACH OF THE FACTORS HAVING CODED LEVEL ZERO.

HIT RETURN TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

FIRST CENTRAL COMPOSITE DESIGNS REQUIRE A TWO (2) LEVEL FULL OR FRACTIONAL DESIGN WHERE NO ONE-FACTOR EFFECTS OR TWO-FACTOR INTERACTIONS CAN BE ALIASED WITH 1, ANOTHER ONE-FACTOR EFFECT, OR ANOTHER TWO FACTOR INTERACTION. HAVE YOU ALREADY DEVELOPED A TWO (2) LEVEL FACTORIAL DESIGN IN THE PROBLEM DEFINITION (SEGMENT 2 - OPTION 1) MEETING THE ABOVE CONDITIONS?  
YES

IF YOU WANT A ROTATABLE CENTRAL COMPOSITE DESIGN, THE CODED LEVEL OF ALPHA WILL BE COMPUTED FOR YOU, OR YOU MAY CHOOSE TO SUPPLY YOUR OWN VALUE FOR THE CODED LEVEL OF ALPHA. DO YOU WANT A ROTATABLE DESIGN?  
YES

THIS IS THE VALUE OF ALPHA NECESSARY FOR A ROTATABLE CENTRAL COMPOSITE DESIGN.

ALPHA = 1.68

HIT RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

TWO METHODS ARE AVAILABLE FOR SPECIFYING REAL WORLD LEVELS.

1. THE FIRST IS TO SPECIFY THE REAL WORLD RANGE THAT YOU WANT FOR EACH FACTOR. THIS WOULD CORRESPOND TO THE CODED LEVELS -ALPHA AND +ALPHA. THIS METHOD

WOULD BE APPROPRIATE WHEN GENERATING A CENTRAL  
COMPOSITE DESIGN FROM SCRATCH.

2. THE SECOND IS TO SPECIFY THE REAL WORLD LEVELS OF  
EACH FACTOR CORRESPONDING TO THE CODED LEVELS -1  
AND 1. THIS METHOD WOULD BE APPROPRIATE IN A  
SEQUENTIAL DESIGN WHERE THE FACTORIAL PART OF THE  
EXPERIMENT HAS ALREADY BEEN COMPLETED.

WHICH METHOD WOULD YOU LIKE? ENTER -- 1 or 2.  
2

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL -1 FOR FACTOR NUMBER 1  
48.24

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL +1 FOR FACTOR NUMBER 1  
101.76

-1 LEVEL = 48.24  
+1 LEVEL = 101.76

ARE THESE THE VALUES THAT YOU WANT?  
YES

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL -1 FOR FACTOR NUMBER 2  
0.18

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL +1 FOR FACTOR NUMBER 2  
0.42

-1 LEVEL = 0.18  
+1 LEVEL = 0.42

ARE THESE THE VALUES THAT YOU WANT?  
YES

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL -1 FOR FACTOR NUMBER 3  
5.98

ENTER THE REAL WORLD VALUE CORRESPONDING TO THE CODED  
LEVEL +1 FOR FACTOR NUMBER 3  
10.02

-1 LEVEL = 5.98  
+1 LEVEL = 10.02

ARE THESE THE VALUES THAT YOU WANT?  
YES

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

--- CENTRAL COMPOSITE (C-C) DESIGN ---

PROBLEM DEFINITION SUMMARY

NO. OF BASIC FACTORIAL LEVELS = 2  
NO. OF BASIC FACTORIAL FACTORS = 3  
NO. OF BASIC FACTORIAL TRIALS = 8  
ALPHA = 1.68

HIT RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

CODED VALUES AND THEIR CORRESPONDING REAL WORLD  
VALUES FOR EACH FACTOR OF THE CENTRAL COMPOSITE DESIGN

	C-C LEVEL	BASE LEVEL	C-C LEVEL	BASE LEVEL	C-C LEVEL
CODED VALUES OF C-C	-1.68	-1.0	0.0	+1.0	1.68
REAL WORLD LEVELS					
FACTOR NO. 1	30.00	48.24	75.00	101.76	120.00
FACTOR NO. 2	0.10	0.18	0.30	0.42	0.50
FACTOR NO. 3	4.60	5.98	8.00	10.02	11.40

HIT RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

TWO-LEVEL BASIC FACTORIAL PORTION OF THE CENTRAL COMPOSITE DESIGN  
(EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO.	1	1	2	3
TRIAL NO.	1	48.24	0.18	5.98
FACTOR NO.	2	1	2	3
TRIAL NO.	2	48.24	0.18	10.02
FACTOR NO.	3	1	2	3
TRIAL NO.	3	48.24	0.42	5.98
FACTOR NO.	4	1	2	3
TRIAL NO.	4	48.24	0.42	10.02

HIT RETURN TO CONTINUE

FACTOR NO. TRIAL NO.	5	1 101.76	2 0.18	3 5.98
FACTOR NO. TRIAL NO.	6	1 101.76	2 0.18	3 10.02
FACTOR NO. TRIAL NO.	7	1 101.76	2 0.42	3 5.98
FACTOR NO. TRIAL NO.	8	1 101.76	2 0.42	3 10.02

HIT RETURN TO CONTINUE

#### CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

#### 6 ADDITIONAL AXIAL POINTS OF THE CENTRAL COMPOSITE DESIGN (EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO. POINT NO.	1	1 120.00	2 0.30	3 8.00
FACTOR NO. POINT NO.	2	1 75.00	2 0.50	3 8.00
FACTOR NO. POINT NO.	3	1 75.00	2 0.30	3 11.40
FACTOR NO. POINT NO.	4	1 30.00	2 0.30	3 8.00

HIT RETURN TO CONTINUE



FACTOR NO.		1	2	3
POINT NO.	5	75.00	0.10	8.00
FACTOR NO.		1	2	3
POINT NO.	6	75.00	0.30	4.60

HIT RETURN TO CONTINUE

# CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

THE CENTER POINT OF THE CENTRAL COMPOSITE DESIGN  
(EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO.	1	2	3
CENTER POINT	75.00	0.30	8.00

THIS CENTRAL COMPOSITE DESIGN IS ROTATABLE,  
IF 6 REPLICATES OF THE CENTER POINT ARE MADE, THE  
VARIANCE OF THE ESTIMATED RESPONSE WILL BE APPROXIMATELY  
THE SAME AT ALL POINTS WITHIN THE SPHERE OF RADIUS ONE.

HIT RETURN TO CONTINUE.

EXAMPLE 2

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

WOULD YOU LIKE TO CHANGE THE RUN I.D.?  
NO

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN  
3

YOUR ENTRY WAS: 3--CENTRAL COMPOSITE DESIGNS

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

AN N-FACTOR CENTRAL COMPOSITE DESIGN CONSISTS OF THREE COMPONENTS.

1. A FULL OR FRACTIONAL REPLICATE OF A TWO-LEVEL FACTORIAL DESIGN, WHERE FOR EACH FACTOR THE TWO CODED LEVELS ARE -1 and 1.
2. TWO CORRESPONDING AXIAL POINTS FOR EACH FACTOR WITH CODED LEVEL -ALPHA AT ONE POINT, +ALPHA AT THE OTHER AND WITH ALL OTHER FACTORS HAVING CODED LEVEL ZERO AT THESE POINTS.
3. THE CENTER POINT OF EACH OF THE FACTORS HAVING CODED LEVEL ZERO.

\*\*\* RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

FIRST CENTRAL COMPOSITE DESIGNS REQUIRE A TWO (2) LEVEL FULL OR FRACTIONAL DESIGN WHERE NO ONE-FACTOR EFFECTS OR TWO-FACTOR INTERACTIONS CAN BE ALIASED WITH 1, ANOTHER ONE-FACTOR EFFECT, OR ANOTHER TWO FACTOR INTERACTION. HAVE YOU ALREADY DEVELOPED TWO (2) LEVEL FACTORIAL DESIGN IN THE PROBLEM DEFINITION (SEGMENT 2 - OPTION 1) MEETING THE ABOVE CONDITIONS?  
NO

THIS REQUIRES THAT THE EXPERIMENTER DESCRIBE A NEW DESIGN. THE MOST EFFICIENT WAY TO DESCRIBE THE NEW DESIGN IS TO USE THE PROGRAM DESIGN DEFINITION CAPABILITY AS IN SEGMENTS 2 - OPTION 1 and 3.

NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS? (2,3,or 5)

2

HOW MANY FACTORS ARE INVOLVED?

YOU MAY CHOOSE UP TO 20.

3

HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?

8

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?

Y

3 FACTORS ARE PRESENT. IS THIS CORRECT?

Y

8 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?

Y

7 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.  
OF THESE:

3 ARE MAIN EFFECTS.

3 ARE FIRST ORDER INTERACTIONS.

1 ARE HIGHER ORDER INTERACTIONS.

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?

THIS MUST BE SELECTED FROM THE VALUES:

2 4 8

8

IS THIS WHAT YOU WANT? 8

Y

THIS IS A FULL FACTORIAL DESIGN.  
FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED  
0 LINEARLY INDEPENDENT DEFINING CONTRAST(s).  
PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

NO ALIASING IS USED IN THIS DESIGN

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
IN THE DATA COLLECTION PROCESS.  
FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
THE LOW AND HIGH FACTOR VALUES.  
WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR TO  
BE PRINTED ON THE LINE PRINTER?  
NO

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABC
000
001
010
011
100
101
110
111

HIT RETURN WHEN READY TO CONTINUE.

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

BASIC FACTORIAL PART OF CENTRAL COMPOSITE DESIGN COMPLETED.

IF YOU WANT A ROTATABLE CENTRAL COMPOSITE DESIGN,  
THE CODED LEVEL OF ALPHA WILL BE COMPUTED FOR YOU,  
OR YOU MAY CHOOSE TO SUPPLY YOUR OWN VALUE FOR THE CODED LEVEL OF ALPHA.  
DO YOU WANT A ROTATABLE DESIGN?  
YES

THIS IS THE VALUE OF ALPHA NECESSARY FOR A ROTATABLE  
CENTRAL COMPOSITE DESIGN.

ALPHA = 1.68

HIT RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

TWO METHODS ARE AVAILABLE FOR SPECIFYING REAL WORLD LEVELS.

1. THE FIRST IS TO SPECIFY THE REAL WORLD RANGE THAT  
YOU WANT FOR EACH FACTOR. THIS WOULD CORRESPOND TO  
THE CODED LEVELS -ALPHA AND +ALPHA. THIS METHOD  
WOULD BE APPROPRIATE WHEN GENERATING A CENTRAL  
COMPOSITE DESIGN FROM SCRATCH.
2. THE SECOND IS TO SPECIFY THE REAL WORLD LEVELS OF  
EACH FACTOR CORRESPONDING TO THE CODED LEVELS -1  
AND 1. THIS METHOD WOULD BE APPROPRIATE IN A  
SEQUENTIAL DESIGN WHERE THE FACTORIAL PART OF THE  
EXPERIMENT HAS ALREADY BEEN COMPLETED.

WHICH METHOD WOULD YOU LIKE? ENTER --1 or 2.

1

ENTER THE LOWER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL -ALPHA FOR FACTOR NUMBER 1  
30.00

ENTER THE UPPER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL +ALPHA FOR FACTOR NUMBER 1.  
120.00

LOWER LEVEL = 30.00  
UPPER LEVEL = 120.00

ARE THESE THE VALUES THAT YOU WANT?  
YES

ENTER THE LOWER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL -ALPHA FOR FACTOR NUMBER 2  
0.10

ENTER THE UPPER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL +ALPHA FOR FACTOR NUMBER 2  
0.50

LOWER LEVEL = 0.10  
UPPER LEVEL = 0.50

ARE THESE THE VALUES THAT YOU WANT?  
YES

ENTER THE LOWER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL -ALPHA FOR FACTOR NUMBER 3  
4.60

ENTER THE UPPER REAL WORLD RANGE CORRESPONDING TO THE CODED  
LEVEL +ALPHA FOR FACTOR NUMBER 3  
11.40



LOWER LEVEL = 4.60  
UPPER LEVEL = 11.40

ARE THESE THE VALUES THAT YOU WANT?  
YES

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

---CENTRAL COMPOSITE (C-C) DESIGN---

PROBLEM DEFINITION SUMMARY

NO. OF BASIC FACTORIAL LEVELS = 2  
NO. OF BASIC FACTORIAL FACTORS = 3  
NO. OF BASIC FACTORIAL TRIALS = 8  
ALPHA = 1.68

HIT RETURN TO CONTINUE

CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

CODED VALUES AND THEIR CORRESPONDING REAL WORLD  
VALUES FOR EACH FACTOR OF THE CENTRAL COMPOSITE DESIGN

	C-C LEVEL	BASE LEVEL	C-C LEVEL	BASE LEVEL	C-C LEVEL
CODED VALUES OF C-C	-1.68	-1.0	0.0	+1.0	1.68
REAL WORLD LEVELS					
FACTOR NO. 1	30.00	48.24	75.00	101.76	120.00
FACTOR NO. 2	0.10	0.18	0.30	0.42	0.50
FACTOR NO. 3	4.60	5.98	8.00	10.02	11.40

HIT RETURN TO CONTINUE

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CENTRAL COMPOSITE DEMONSTRATION  
\*\*\*\*\*

TWO-LEVEL BASIC FACTORIAL PORTION OF THE CENTRAL COMPOSITE DESIGN  
(EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO. TRIAL NO.	1	1	2	3
		48.24	0.18	5.98
FACTOR NO. TRIAL NO.	2	1	2	3
		48.24	0.18	10.02
FACTOR NO. TRIAL NO.	3	1	2	3
		48.24	0.42	5.98
FACTOR NO. TRIAL NO.	4	1	2	3
		48.24	0.42	10.02

HIT RETURN TO CONTINUE

FACTOR NO. TRIAL NO.	5	1 101.76	2 0.18	3 5.98
FACTOR NO. TRIAL NO.	6	1 101.76	2 0.18	3 10.02
FACTOR NO. TRIAL NO.	7	1 101.76	2 0.42	3 5.98
FACTOR NO. TRIAL NO.	8	1 101.76	2 0.42	3 10.02

HIT RETURN TO CONTINUE

# CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

## 6 ADDITIONAL AXIAL POINTS OF THE CENTRAL COMPOSITE DESIGN (EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO. POINT NO.	1	1 120.00	2 0.30	3 8.00
FACTOR NO. POINT NO.	2	1 75.00	2 0.50	3 8.00
FACTOR NO. POINT NO.	3	1 75.00	2 0.30	3 11.40
FACTOR NO. POINT NO.	4	1 30.00	2 0.30	3 8.00

HIT RETURN TO CONTINUE

FACTOR NO.		1	2	3
POINT NO.	5	75.00	0.10	8.00

FACTOR NO.		1	2	3
POINT NO.	6	75.00	0.30	4.60

HIT RETURN TO CONTINUE

# CENTRAL COMPOSITE DEMONSTRATION

\*\*\*\*\*

THE CENTER POINT OF THE CENTRAL COMPOSITE DESIGN  
(EXPRESSED IN REAL WORLD LEVELS)

FACTOR NO.		1	2	3
CENTER POINT		75.00	0.30	8.00

THIS CENTRAL COMPOSITE DESIGN IS ROTATABLE,  
IF 6 REPLICATES OF THE CENTER POINT ARE MADE, THE  
VARIANCE OF THE ESTIMATED RESPONSE WILL BE APPROXIMATELY  
THE SAME AT ALL POINTS WITHIN THE SPHERE OF RADIUS ONE.

HIT RETURN TO CONTINUE.

### ACTUAL EXPERIMENTAL DESIGN

Segment 3--Actual Experimental Design--uses the previously specified experimental definition (Segment 2 - Option 1) to construct the set of experimental treatments to be run. This set is called the basic experimental block/observation vectors. To aid the experimenter in deciding if the experiment has an acceptable structure, an alias summary is displayed that shows how main effects and first-order interactions are aliased. The user may also have the aliasing of any specific effect displayed.

If the design is unacceptable, the experimenter can rerun the problem definition phase.

The following is an example of the experimental design of the problem defined in Segment 2 - Option 1 - Example 1.

ACTUAL EXPERIMENTAL DESIGN

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
3

YOUR ENTRY WAS: 3--ACTUAL EXPERIMENTAL DESIGN

WOULD YOU LIKE TO CHANGE THE RUN I.D.?  
YES

PLEASE ENTER THIS RUN PROBLEM I.D.  
ACTUAL EXPERIMENTAL DESIGN DEMONSTRATION

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS  
AND FIRST ORDER INTERACTIONS CONFOUNDED WITH  
EACH MAIN EFFECT AND FIRST ORDER INTERACTION.  
MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE  
MAIN EFFECTS THAT ARE CONFOUNDED WITH OTHER  
MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE

#### EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	1 M	A
0	0	1	AB
0	0	1	AC
0	0	1	AD

0	0	1	AE
0	0	1 M	B
0	0	1	BC
0	0	1	BD
0	0	1	BE
0	0	1 M	C
0	0	1	CD
0	0	1	CE
0	0	1 M	D
0	0	1	DE
0	0	1 M	E

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE.

If any of the main effects had been confounded with another main effect, it would have been flagged with an \*.

Note that in this design, none of the main effects or first-order interactions are confounded with any other main effects or first-order interactions.

# ACTUAL EXPERIMENTAL DESIGN DEMONSTRATION

\*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?

ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.

EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

A

BCDE

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?

ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.

EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB

CD

ABE

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?

ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.

EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.



BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
 THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
 IN THE DATA COLLECTION PROCESS.  
 FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
 THE LOW AND HIGH FACTOR VALUES.

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR  
 TO BE PRINTED ON THE LINE PRINTER?  
 N

HIT RETURN WHEN READY TO CONTINUE.

ACTUAL EXPERIMENTAL DESIGN DEMONSTRATION

\*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABCDE
00000
00011
00101
00110
01001
01010
01100
01111
10001
10010
10100
10111
11000
11011
11101
11110

HIT RETURN WHEN READY TO CONTINUE.

### DATA ANALYSIS

Once the data have been collected, it must be analyzed to identify significant effects. This analysis could consist of an analysis of variance (ANOVA) or of a regression analysis.

The capability to perform the data analysis has not been included in the AED program at this time since a program to analyze a fractional factorial experiment with both regression analysis and ANOVA techniques would be a full-time project in itself. The program user should analyze his data with existing routines available at AMRL or other computer facilities.

### EXPERIMENTAL REFINEMENT

Once the experimenter has conducted a fraction of a full factorial experiment, an analysis of the basic block data may provide sufficient information to preclude additional data collection. If one or more factors produce significant results, all further work may be confined to studying these factors in detail. The experiment may be redesigned with fewer factors or with other factors added. Additional work is required if:

1. The main effects are not given with sufficient precision.
2. Some main effects may be confounded with two-factor interactions and may require separation.
3. Some two-factor interactions may require separation.
4. Additional factors may need to be included in the design.

### SEPARATION OF ALIASES

Separation of aliases assumes that a fractional design has been executed and that we wish to collect additional data in order to obtain a higher degree of precision and/or to separate the main effects from two-factor interactions.

A  $1/2$  factorial or a  $1/3$  factorial will require a full factorial to separate aliases. Thus, this assumes that a  $1/4$  or  $1/9$  (etc.) fractional factorial

has been performed and the problem is to separate effect X from effect Y, i.e., X and Y are in the same aliasing group,  $X=Y$ .

The user will define the aliased terms and the program will tell the user which of his original alias terms may be deleted to remove this aliasing.

The following is an example of the separation of alias feature.

The problem can be defined in one of two ways. The user may use program segments 2 - options 1 and 3 to define the problem, then enter segment 4 to perform the refinement (see example 1).

Or the user may enter directly into segment 4 in which case, the program will prompt him for the program definition (see example 2).

EXPERIMENTAL REFINEMENT

EXAMPLE 1

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
2

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBLEM I.D.  
EXPERIMENTAL REFINEMENT DEMONSTRATION

EXPERIMENTAL REFINEMENT DEMONSTRATION  
\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN  
1

YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS

HOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE  
ANALYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS? (2,3, or 5)  
2

HOW MANY FACTORS ARE INVOLVED?  
YOU MAY CHOOSE UP TO 20.  
6

HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?

20

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?

Y

6 FACTORS ARE PRESENT. IS THIS CORRECT?

Y

20 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?

Y

63 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.

OF THESE:

6 ARE MAIN EFFECTS

15 ARE FIRST ORDER INTERACTIONS

42 ARE HIGHER ORDER INTERACTIONS

HIT RETURN WHEN READY TO CONTINUE

This is an example of using segments 2 - option 1 and 3 to define the problem.

EXPERIMENTAL REFINEMENT DEMONSTRATION

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?

THIS MUST BE SELECTED FROM THE VALUES?

2	4	8	16	32	64
---	---	---	----	----	----

16

IS THIS WHAT YOU WANT: 16

Y

THIS IS A 1/4 FRACTIONAL FACTORIAL DESIGN.

FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED 2 LINEARLY INDEPENDENT DEFINING CONTRAST(s).

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A, B, C, D, ETC.  
THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES THE FACTOR IS AT ITS LOW LEVEL.  
THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

AC MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL,  
AND FACTOR C AT ITS HIGH LEVEL.  
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
NO.

YOU MAY DEFINE CONTRASTS BY DESCRIBING WHICH EFFECTS ARE TO BE CONFOUNDED.  
FOR EXAMPLE, AB-CD. ANOTHER (MORE COMMON) WAY IS TO DEFINE THE ALIASING  
IN TERMS OF THE IDENTITY EFFECT. FOR EXAMPLE, I=ABCD.

REMEMBER THAT THE DEFINING CONTRAST MUST BE OF THE FORM EFFECT 1 - EFFECT 2  
THE FIRST CONTRAST EFFECT IS INPUT THEN THE SECOND.  
DEFINING CONTRAST NO. 1 FIRST TERM

I  
DEFINING CONTRAST NO. 1 SECOND TERM  
ABCD

IS THIS CORRECT?

I - ABCD

Y

DEFINING CONTRAST NO. 2 FIRST TERM

I

DEFINING CONTRAST NO. 2 SECOND TERM

ACDEF

IS THIS CORRECT?

I - ACDEF

Y

This is an example of how the alias set is input if the predefined (stored) alias set was unacceptable or unavailable.



THE DEFINING CONTRAST SET DEFINED CONTAINS 2 INDEPENDENT MEMBER(s).

DO YOU WANT TO SEE THE TOTAL ALIAS SET?  
YES

TOTAL ALIAS SET  
ACDEF  
ABDC  
BEF

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?  
NO

PROBLEM DEFINITION COMPLETED.

HIT RETURN WHEN READY TO CONTINUE

EXPERIMENTAL REFINEMENT  
\*\*\*\*\*

YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS:

1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN

4

YOUR ENTRY WAS: 4--EXIT

WELCOME TO THE SCREENING DESIGNS PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN

3

YOUR ENTRY WAS: 3--ACTUAL EXPERIMENTAL DESIGN.

WOULD YOU LIKE TO CHANGE THE RUN I.D.?

NO

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS AND FIRST ORDER INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER INTERACTION. MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE MAIN EFFECTS THAT ARE CONFOUNDED WITH OTHER MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	3 M	A
0	1	2	AB
0	1	2	AC
0	1	2	AD
0	0	3	AE
0	0	3	AF
0	1	2 M	B

0	1	2	BC
0	1	2	BD
1	0	2	BE
1	0	2	BF
0	0	3 M	C
0	1	2	CD
0	0	3	CE
0	0	3	CF
0	0	3 M	D
0	0	3	DE
0	0	3	DF
0	1	2 M	E

HIT RETURN WHEN READY TO CONTINUE

1	0	2	EF
0	1	2 M	F

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT DEMONSTRATION

\*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

A

ABEF  
 CDEF  
 BCD

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
 THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
 IN THE DATA COLLECTION PROCESS.  
 FOR A 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
 THE LOW AND HIGH FACTOR VALUES. FOR A 3 LEVEL PER FACTOR  
 EXPERIMENT, 0, 1, AND 2 REPRESENT THE LOW, INTERMEDIATE,  
 AND HIGH FACTOR VALUES.

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT DEMONSTRATION

\*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABCDEF
000000
000011
001100
001111
010101
010110
011001
011010
100100
100111
101000
101011
110001
110010
111101
111110

HIT RETURN WHEN READY TO CONTINUE.  
 EXPERIMENTAL REFINEMENT DEMONSTRATION

\*\*\*\*\*

Note that effect A is confounded with effect ABEP. The user may want to use segment 5 to separate these two effects.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
4

YOUR ENTRY WAS: 4--EXPERIMENTAL REFINEMENT

WOULD YOU LIKE TO CHANGE THE RUN I.D.?  
NO

EXPERIMENTAL REFINEMENT DEMONSTRATION  
\*\*\*\*\*

I HAVE THE BASIC EXPERIMENTAL BLOCK AS IN THE ORIGINAL PROBLEM.  
NOW, YOU MUST SPECIFY THE TWO ALIASED EFFECTS  
YOU WISH TO HAVE SEPARATED.

WHAT IS THE FIRST EFFECT:

A

WHAT IS THE SECOND EFFECT:

ABEF

ALIASED EFFECT: A

ALIASED EFFECT: ABEF

ARE THESE THE ALIASED EFFECTS?

YES

HIT RETURN WHEN READY TO CONTINUE.

MAY DELETE ALIAS NUMBER 1: 1=ABCD  
 MAY DELETE ALIAS NUMBER 2: 1=ACDEF

WHICH ALIAS NUMBER WOULD YOU LIKE TO ELIMINATE?  
 2

The separation can be accomplished by deleting either alias number 1 or number 2. In this case, number 2 is chosen.

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS AND FIRST ORDER INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER INTERACTION. MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE EFFECTS THAT ARE CONFOUNDED WITH OTHER MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE

# EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	1 M	A
0	1	0	AB
0	1	0	AC
0	1	0	AD
0	0	1	AE
0	0	1	AF
0	0	1 M	B
0	1	0	BC
0	1	0	BD
0	0	1	BE
0	0	1	BF
0	0	1 M	C
0	1	0	CD
0	0	1	CE
0	0	1	CF
0	0	1 M	D
0	0	1	DE
0	0	1	DF
0	0	1	E

HIT RETURN WHEN READY TO CONTINUE

0	0	1	EF
0	0	1	F

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE

EXPERIMENTAL REFINEMENT DEMONSTRATION

\*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

A

BCD

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

Note: Effect A is no longer confounded with AB, EF.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
IN THE DATA COLLECTION PROCESS.  
FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
THE LOW AND HIGH FACTOR VALUES.  
NEW TRIALS (NOT PART OF PREVIOUS BLOCK)  
ARE MARKED WITH THE LETTER N

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR  
TO BE PRINTED ON THE LINE PRINTER?

NO

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT DEMONSTRATION  
 \*\*\*\*\*  
 BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABCDEF	
000000	
000001	N
000010	N
000011	
001100	
001101	N
001110	N
001111	
010100	N
010101	
010110	
010111	N
011000	N
011001	
011010	
011011	N
100100	
100101	N
100110	N
100111	

HIT RETURN WHEN READY TO CONTINUE.



	ABCDEF
N	101001
N	101010
	101011
N	110000
	110001
	110010
N	110011
N	111100
	111101
	111110
N	111111

HIT RETURN WHEN READY TO CONTINUE.

Assuming that the previous block of experiments had already been performed, only those trials marked with an N (new) need to run now.

EXAMPLE 2

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN  
4

YOUR ENTRY WAS: 4--EXPERIMENTAL REFINEMENT

PLEASE ENTER THIS RUN PROBLEM I.D.  
EXPERIMENTAL REFINEMENT

This is an example of entering directly into segment number 4, without first entering segment 2 - option 1.

EXPERIMENTAL REFINEMENT  
\*\*\*\*\*

THIS REQUIRES THAT THE EXPERIMENTER DESCRIBE THE ORIGINAL DESIGN.  
THE MOST EFFICIENT WAY TO DESCRIBE THE ORIGINAL DESIGN IS TO USE THE PROGRAM  
DESIGN DEFINITION CAPABILITY AS IN SEGMENTS 2 - OPTION 1 and 3.

YOU MUST SPECIFY THE DESIGN IN THE SAME WAY AS IN THE ORIGINAL RUN.

NOW WE MUST DEFINE THE SPECIFIC PROBLEM  
TO BE ANALYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS? (2,3, or 5)  
2

HOW MANY FACTORS ARE INVOLVED?  
YOU MAY CHOOSE UP TO 20.  
6

HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?  
16

2 LEVELS FOR THE FACTORS. IS THIS CORRECT?  
Y

6 FACTORS ARE PRESENT. IS THIS CORRECT?  
Y

16 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?  
Y

63 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS. OF THESE:  
6 ARE MAIN EFFECTS.  
15 ARE FIRST ORDER INTERACTIONS.  
42 ARE HIGHER ORDER INTERACTIONS.

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT

\*\*\*\*\*

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?  
THIS MUST BE SELECTED FROM THE VALUES:

2 4 8 16 32 64  
16

IS THIS WHAT YOU WANT: 16  
Y

THIS IS A 1/4 FRACTIONAL FACTORIAL DESIGN.  
FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED 2 LINEARLY INDEPENDENT  
DEFINING CONTRAST (S).

THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A, B, C, D, ETC.  
THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO  
DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES  
THE FACTOR IS AT ITS LOW LEVEL.  
THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.

AC MEANS FACTOR A AT ITS HIGH LEVEL, FACTOR B AT ITS LOW LEVEL,  
AND FACTOR C AT ITS HIGH LEVEL.  
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?  
NO

YOU MAY DEFINE CONTRASTS BY DESCRIBING WHICH EFFECTS ARE TO BE CONFOUNDED.  
FOR EXAMPLE, AB-CD. ANOTHER (MORE COMMON) WAY IS TO DEFINE THE ALIASING  
IN TERMS OF THE IDENTITY EFFECT. FOR EXAMPLE, I-ABCD.

REMEMBER THAT THE DEFINING CONTRAST MUST BE OF THE FORM EFFECT 1 - EFFECT 2  
THE FIRST CONTRAST EFFECT IS INPUT THEN THE SECOND.

DEFINING CONTRAST NO. 1 FIRST TERM  
I

DEFINING CONTRAST NO. 1 SECOND TERM  
ABCD

IS THIS CORRECT?

I -ABCD  
Y

DEFINING CONTRAST NO. 2 FIRST TERM  
I

DEFINING CONTRAST NO. 2 SECOND TERM  
ACDEF

IS THIS CORRECT?

I -ACDEF  
Y

THE DEFINING CONTRAST SET DEFINED CONTAINS 2 INDEPENDENT MEMBER(s).

DO YOU WANT TO SEE THE TOTAL ALIAS SET?  
YES

TOTAL ALIAS SET  
ACDEF  
ABCD  
BEF

HIT RETURN WHEN READY TO CONTINUE.

WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?  
NO

EXPERIMENTAL REFINEMENT

\*\*\*\*\*

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS  
AND FIRST ORDER INTERACTIONS CONFOUNDED WITH  
EACH MAIN EFFECT AND FIRST ORDER INTERACTION.  
MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE  
MAIN EFFECTS THAT ARE CONFOUNDED WITH OTHER  
MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE.

# EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	3 M	A
0	1	2	AB
0	1	2	AC
0	1	2	AD
0	0	3	AE
0	0	3	AF
0	1	2 M	B
0	1	2	BC
0	1	2	BD
1	0	2	BE
1	0	2	BF
0	0	3 M	C
0	1	2	CD

0	0	3	CE
0	0	3	CF
0	0	3 M	D
0	0	3	DE
0	0	3	DF
0	1	2 M	E

HIT RETURN WHEN READY TO CONTINUE.

1	0	2	EF
0	1	2 M	F

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT  
 \*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

A  
 CDEF  
 BCD  
 ABEF

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
 THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
 IN THE DATA COLLECTION PROCESS.  
 FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
 THE LOW AND HIGH FACTOR VALUES.

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR TO BE  
PRINTED ON THE LINE PRINTER?

I DID NOT UNDERSTAND YOUR RESPONSE.  
PLEASE ANSWER THE QUESTION WITH A YES OR NO RESPONSE.  
NO.

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT

\*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

ABCDEF  
000000  
000011  
001100  
001111  
010101  
010110  
011001  
011010  
100100  
100111  
101000  
101011  
110001  
110010  
111101  
111110

HIT RETURN WHEN READY TO CONTINUE.



EXPERIMENTAL REFINEMENT  
\*\*\*\*\*

I HAVE THE BASIC EXPERIMENTAL BLOCK AS IN THE ORIGINAL PROBLEM.  
NOW, YOU MUST SPECIFY THE TWO ALIASED EFFECTS YOU WISH TO  
HAVE SEPARATED.

WHAT IS THE FIRST EFFECT:

A

WHAT IS THE SECOND EFFECT:

ABEF

ALIASED EFFECT: A

ARE THESE THE ALIASED EFFECTS?

YES

HIT RETURN WHEN READY TO CONTINUE

MAY DELETE ALIAS NUMBER 1: I-ABCD

MAY DELETE ALIAS NUMBER 2: I-ACDEF

WHICH ALIAS NUMBER WOULD YOU LIKE TO ELIMINATE?

2

THIS IS A SUMMARY OF THE NUMBER OF MAIN EFFECTS  
AND FIRST ORDER INTERACTIONS CONFOUNDED WITH  
EACH MAIN EFFECT AND FIRST ORDER INTERACTION.  
MAIN EFFECTS ARE MARKED WITH THE LETTER M, AND THOSE  
MAIN EFFECTS THAT ARE CONFOUNDED WITH OTHER  
MAIN EFFECTS ARE FLAGGED WITH AN \*.

HIT RETURN WHEN READY TO CONTINUE.

# EXPERIMENTAL DESIGN SUMMARY

MAIN	1-ST	HIGHER	EFFECT
0	0	1 M	A
0	1	0	AB
0	1	0	AC
0	1	0	AD
0	0	1	AE
0	0	1	AF
0	0	1 M	B
0	1	0	BC
0	1	0	BD
0	0	1	BE
0	0	1	BF
0	0	1 M	C
0	1	0 <sup>1</sup>	CD
0	0	1	CE
0	0	1	CF
0	0	1 M	D
0	0	1	DE
0	0	1	DF
0	0	1 M	E

HIT RETURN WHEN READY TO CONTINUE.

0	0	1	EF
0	0	1 M	F

END OF SUMMARY TABLE

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT  
 \*\*\*\*\*

WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.  
 A

BCD  
 WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?  
 ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.  
 EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR  
 THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN  
 IN THE DATA COLLECTION PROCESS.  
 FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT  
 THE LOW AND HIGH FACTOR VALUES.  
 NEW TRIALS (NOT PART OF PREVIOUS BLOCK)  
 ARE MARKED WITH THE LETTER N.

WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR  
 TO BE PRINTED ON THE LINE PRINTER?  
 NO

HIT RETURN WHEN READY TO CONTINUE.

EXPERIMENTAL REFINEMENT  
 \*\*\*\*\*

BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

	ABCDEF
	000000
N	000001
N	000010
	000011
	001100
N	001101
N	001110
	001111

N	010100
	010101
	010110
N	010111
N	011000
	011001
	011010
N	011011
	100100
N	100101
N	100110
	100111

HIT RETURN WHEN READY TO CONTINUE.

ABCDEF	101000
N	101001
N	101010
	101011
N	110000
	110001
	110010
N	110011
N	111000
	111011
	111110
N	111111

HIT RETURN WHEN READY TO CONTINUE.

EXIT

Segment 5--Exit is the segment to enter from the AED program.

The following is an example of its use.

EXIT

WELCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.

YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS:

1. BASIC TERMINOLOGY
2. PROBLEM DEFINITION
3. ACTUAL EXPERIMENTAL DESIGN
4. EXPERIMENTAL REFINEMENT
5. EXIT

ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN

5

YOUR ENTRY WAS: 5--EXIT PROGRAM

ROUTINE WRAPUP ENTERED --

NO INTERNAL PROCESSING REQUIRED BY THIS SYSTEM.

FORTRAN STOP

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